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Teachers' knowledge of children's mathematical development

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ABSTRACT

This research was conducted due to the importance value of mathematics for early childhood and the fact some researches showed the early childhood education (ECE) teachers' low level of basic mathematical knowledge, especially the one related to childhood developmental stages. The participants of this research were 35 ECE teachers from one of the cities in West Java province with teaching experience approximately ten years. In this research, 30 minutes was given to the participants to solve 20 questions, which tested teachers' knowledge related to verbal counting sequence, counting, the ordinal number of words, addition/subtraction, divisions of sets, written number symbols, and words. Besides, the interview was conducted to get more in-depth information from the participants. The quantitative descriptive analysis was used to identify the frequency, percentage, mean value, and standard deviation. The result of the research showed that ECE teachers had limited knowledge of children's mathematical development. It was revealed by the result of the mean value of the teachers' responses, which were only 33% correct answers and 16% no idea answers. This result can become input for the stakeholders to hold a professional development program which aims to increase the quality of ECE teachers related to mathematical development activity.

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1. INTRODUCTION

This research was conducted due to the importance value of early childhood in learning mathematics. Mathematics at early childhood is essential because it provides a strong foundation for children to learn in the future [1-8]. Therefore, children are expected to have the ability to learn and understand any concepts of mathematics [1, 9-13]. NCTM [7] emphasized that good quality, challenging, and easily accessible to mathematical education for children age 3 to 6 is considered an essential foundation for mathematical learning in the future.

On the other hand, teachers have a significant role in building children's mathematical development. According to Vygotsky [14], the teachers' role is as The More Knowledgeable Other (MKO). MKO refers to anyone who has a higher ability than the students; teachers, peers, or computers [15-20]. Vygotsky added that MKO could be considered as a model, someone who is trusted by the children and whom they found some experience through interaction [14-23]. To sum up, the teachers' knowledge is a very significant asset in applying any developmental activities in the classrooms.

However, there is a problem related to teachers' knowledge. It is said that teachers' knowledge of children's mathematical development is less satisfying [24, 25]. The teachers admitted that when teaching, they

were never aware of children's developmental stages [25, 26]. NRC [5] supported the result of the research, which stated that a lot of ECE teachers ignored and never considered students' initial abilities before they started learning. Identifying initial ability is an essential stage in children's mathematical knowledge acquisition [24-26]. When teachers are motivated to use their informal knowledge about students' thinking in more meaningful ways, it helps in improving students' deeper understanding from the strategy of their students' own thinking [27]. It is proven that focusing on students' individual thinking brings a positive impact on the classroom learning process [28].

The objective of this research is to investigate ECE teachers' knowledge of children's mathematical development. The concept which was examined was the concept of number sense only. Charlesworth stated that number sense is an intuitive sense towards numbers and the usage as well as their various interpretations. Number sense is one of the most fundamental concepts to be developed at the beginning stage of scientific development activities [29, 30]. It is not just a foundation, but it also affects children's mathematical achievement in primary school [29]. It means that children with a strong ability of number sense at early childhood are indicated to make higher mathematical achievement in primary school. Therefore, teachers are expected to improve their quality in order to present excellent developmental activities in number sense.

2. THEORETICAL FRAMEWORK

The theory referred by this research was Mathematical Knowledge for Teaching (MKT). The concept of MKT was taken from the note of mathematics teaching and teachers' repeated task identification, which needs knowledge, reasoning, and mathematics knowledge [31-37]. Ball [36] stated that a teacher must at least have two types of mathematics knowledge; SMK as knowledge related to content knowledge and PCK as knowledge of how to impart mathematics. The relationship between PCK and SMK can be seen in Figure 1.

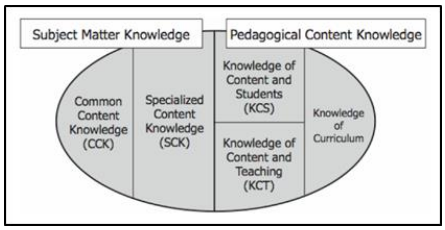

Figure 1. The domain of mathematical knowledge for teaching [34]

The domain related to this research is Knowledge of Content and Students (KCS), which is a part of Pedagogical Knowledge for Teaching (PCK). In this context, PCK generally refers to the ability to teach mathematics effectively [31-37], while KCS is a combination of knowledge about students and knowledge about students' mathematics [36, 37]. Ball added that students' conception skill and general misconception regarding certain content in mathematics are included in the components of KCS. Besides, KCS is also an ability in predicting and analyzing students' processes of thinking [31, 38, 39]. Teachers use KCS to predict students' responses and to anticipate what to be done in developmental activities.

Furthermore, this research was also referred to Vygotsky's theory. In terms of definition, Vygotsky's theory is a part of a branch of wide-ranging constructivism theory [14-23]. The discussion of Vygotsky's theory is centered more on the argument saying that the social relationship between people and culture develops one's knowledge [16, 19, 23]. Some concepts of Vygotsky's theory which are in line with this research are (1) social interaction plays an essential role in the process of cognitive development; (2) The More Knowledgeable Other (MKO); (3) Zone of Proximal Development (ZPD), and 4) Scaffolding. The following is the elaboration of these concepts.

The implication of Vygotsky's theory upon Knowledge of Content and Students (KCS) is that teachers as More Knowledgeable Other (MKO) are able to plan the best way of how mathematics concept is given through accurate stimuli in accordance with the children's characteristics so that the obstacles in learning can be minimalized [17]. The stimuli are in the form of social interaction and scaffolding. These two concepts are given to the children after MKO understood children's abilities [40, 41]. The two concepts are also considered as a bridge to connect what the children have known before with something new, and the children will learn [42]. Teachers, as MKO, should have the ability to anticipate how students would think through an assignment of a developmental activity [7].

3. RESEARCH METHOD

3.1. Participants

Participants of this research were 35 ECE teachers in one of the cities in West Java Province who have ten year-teaching experience. The decision of 10 year-teaching experience was made because 10 year-teaching experience in ECE indicated that teachers are able to interpret and improve children's mathematical thinking [3, 43, 44]. All participants were scholars of Early Childhood Education who were selected purposively.

3.2. Data collection and analysis procedure

At this stage, the analysis of teachers' knowledge about the process and stages of children's mathematical development was conducted by solving questions. Thirty minutes was given to solve 20 questions to test teachers' knowledge about verbal counting sequence, counting, the ordinal number of words, addition/subtraction, divisions of sets, written number symbols, and words. The questions was based on the instrument of Knowledge Mathematical Development by Kim [25, 26]. The following is an example of the questions. Table 1 shows the examples of problems.

Of the two statements below, choose which mathematics skill a child should possess first. If both skills could be acquired at one time, choose "the same." If you don't know the answer, choose "no idea."

Table 1. Example of problems

No	Choice (√)	Statements
13		Amri is dealing with two groups of buttons. One group consists of five, and the other consists of two.  When asked, "how many buttons altogether?" Amri started to count all buttons starting from the first five buttons of one group. **Amri: "one, two, three, four, five, six, seven." There are seven buttons.** Amri is dealing with two groups of buttons. One group consists of five, and the other consists of two.  When asked, "how many buttons altogether?" Amri started to count from the total number of one group. **Amri: "five, six, seven." There are seven buttons.** The same No idea

In each question, participants had to choose which of two mathematics skills a child should possess first. If the participants thought that both skills have an equal level of difficulty, then they would select "the same." Meanwhile, if the participants had no idea which skill is more natural for children to learn earlier, then they would select "no idea."

In addition to giving questions, interviews were also conducted to several participants to follow up on answers to the questions they were working on. The interview was conducted randomly based on the results of the answers to the participants' questions.

The quantitative descriptive analysis was used to identify the frequency, percentage, mean value, and standard deviation. A descriptive statistic is used to describe the characteristics of a particular sample of individuals and is considered a fundamental part of quantitative research [45]. The participants' responses were coded to imply the correct answer by the recommendation of the instrument writer [25]. In this research, the evaluation was based on the number of correct sequences. The total score was calculated from the teachers' correct answers.

4. RESULTS AND ANALYSIS

The result of this research showed that 33% of the participants responded well to questions regarding teachers' knowledge of developmental stages and children's mathematics process. The best result was that 85% of the participants able to give correct answer to the following questions in Table 2.

Table 2. Question number 4

No	Choice (√)	Statements
4		Angel matches seven forks "one-to-one" with seven plates. Angel counts a row of 7 forks.  The same No idea

However, three questions received a less satisfying number of percentages; they were only 8 % correct answer. The questions were related to these topics; Teagan is answering/solving addition questions, Sage is counting a series of 7 buttons, and Indigo can answer, "show the second sheep that is lining now." Table 3 below are the teachers' results of questions about mathematical development.

Table 3. Results of questions about mathematical development

Question Topic	Correct Answer	
	f	*%*
Angel matches seven forks	11	85%
Jaiden counts a row of six buttons	9	69%
Pauli counts a group of seven buttons	7	54%
Sam says the counting words	6	46%
Shea answered a question when the teacher brought her two groups of dolls	6	46%
Kaiden says the counting words in order from 1 to 6	6	46%
Micah counts from 1 to 6	5	38%
Kim divides 12 cookies between two puppets equally	5	38%
Jamie says the counting words in order from 1 to 10	4	31%
Angel counts a row of 8 teddy bears	4	31%
Peyton counts a row of 8 buttons	4	31%
Amari starts to count all the buttons	3	23%
Justine writes one digit of the number	3	23%
Daevon reads a digit of the number	3	23%
Cimarron counts a row of 10 buttons	2	15%
Cyprus answers the question, "what is five plus one?"	2	15%
Pilar counts a row of 7 buttons.	2	15%
Teagan answers addition questions	1	8%
Sage counts a row of 7 buttons.	1	8%
Indigo answers the question, “point to the second sheep that is lining now”.	1	8%
Mean Value		**33%**

Table 4 shows the data of those who answered “no idea” to the questions of the test. The mean value of those who responded “no idea” was 16 %.

Table 4. The answer to "no idea."

Question Topic	“No idea” Answer	
	f	*%*
Teagan answers addition questions	*8*	*47%*
Angel counts a row of 8 teddy bears	7	41%
Shea answered a question when the teacher brought her two groups of dolls	7	41%
Kim divides 12 cookies between two puppets equally	6	35%
Peyton counts a row of 8 buttons	5	29%
Daevon reads one digit of the number	4	24%
Micah counts from 1 to 6	3	18%
Amari starts to count all the buttons	3	18%
Indigo answers the question, "point to the second sheep that is lining now."	3	18%
Jaiden counts a row of 6 buttons	2	12%
Cyprus answers the question, "what is five plus one?"	2	12%
Pilar counts a row of 7 buttons	2	12%
Justine writes one digit of the number	2	12%
Kaiden says the counting words in order from 1 to 6	1	6%
Sam says the counting words	0	0%
Jamie says the counting words in order from 1 to 10	0	0%
Pauli counts a group of 7 buttons	0	0%
Angel matched seven forks	0	0%
Cimarron counts a row of 10 buttons	0	0%
Sage counts a row of 7 buttons	0	0%
Mean Value		**16%**

According to the result of the interview, the participants said that they had never considered the children's developmental stages while they were teaching, as mentioned in the questions. Some participants had known the developmental stages from some resources but not that much detail. However, most of the participants stated that they had never learned the knowledge or information regarding children's developmental stages. The response of the participants implied that they found it challenging to identify the children's mathematical development stages.

5. DISCUSSION

As previously mentioned, this research was to measure what teachers know about children's mathematical development. The result of this research showed that the result of the mean value of the teacher's responses, which were only 33% correct answers and 16% no idea answers. This finding implied that teachers' knowledge of mathematical development was limited. Furthermore, according to the interview, most participants stated that they had not received any information regarding children's developmental stages. In fact, in order to teach ECE children mathematics, teachers need to be equipped with the skill, education, and right collaborative experiences, which can include modeling, classroom observation, coaching, and study groups [46]; and children's developmental stage [11, 27].

The result of this research is in line with the research of Kim [26], which stated the lack of teachers' knowledge of children's mathematical development. Kim stated that preservice and in-service teachers' knowledge of children's development is somewhat limited. Teachers' knowledge of mathematical development determines their ability to make a significant impact on the development of mathematical concepts and goals in young children [25, 26]. If teachers know what conception and misconception the children might experience when they were developed by mathematical content, the teachers would be more ready facing any obstacles in front of them [24]. If teachers are motivated to use their knowledge about students' thinking in more meaningful ways, it helps in improving students' understanding because they used students' thinking strategies [27]. Teachers as The More Knowledgeable Other (MKO) need to have the ability to anticipate how students are thinking through the task of a developmental activity [7, 8, 47].

The limited ECE teachers' knowledge problem is not a new finding, especially for researchers of ECE teachers. Some researchers reported that most higher education and universities do not offer much information related to the topic, or equipped teachers' education with the appropriate skill to teach mathematics to children [3, 4]. Education for early childhood teachers offers some or even no requirement of mathematics education [1, 25]. It makes teachers have a low level of education, without enough preparation to teach mathematics [1]. Some early childhood programs do not focus on the high quality of mathematics instruction, even though many researchers supported theory, which stated that early mathematics experience could affect the outcome of mathematics education in the future [10-13, 48].

Furthermore, the result of this research contributed as input for the stakeholders to increase the quality of ECE teachers. Considering that all participants were graduated from the ECE education program, so the result of this research can be used as a suggestion for the implementation of professional development activity. Professional development is significant for career development as well as to adapt relevant changes in ECE. Professional development is a process and activity which is designed and held to improve knowledge of professional development, skill, and teachers' competencies, which later would improve the students' knowledge [49, 50]. Professional development programs must be arranged by considering research-based knowledge of students and teachers [24]. Professional development program needs to provide opportunities for teachers to work in the framework of research-based work to understand practical teaching rather than just learning new strategies to solve some specific problem [33].

In this research, the participants need professional development to improve their knowledge of children's mathematical development. ECE teachers must understand how children learn mathematical knowledge by evaluating their understanding and observing their progress [10, 12, 25, 26]. Also, it is essential for the teachers to know how to support mathematical development of the children in the classroom, to develop effective and appropriate classroom learning, and to present high quality of mathematics teaching for early childhood.

6. CONCLUSION

The result of this research showed that ECE teachers have limited knowledge of children's mathematical development. It was revealed by the result of the mean value of the teacher's responses, which were only 33% correct answers and 16% no idea answers. The participants' reasons were they 1) had never considered children's developmental stages while they were teaching; 2) knew developmental stages from some resources but not that much detailed; 3) had never received knowledge or information regarding children's developmental stages. Information provided by this research has implications upon the quality of ECE teachers in introducing mathematics to the children, considering the importance of children develop knowledge and developmental activity. In order to promote mathematical knowledge development and to give high quality of mathematical instruction at ECE, teachers must understand young children's cognitive, physical, social, emotional, and their development. In this case, teachers must have the knowledge and adequate skill of mathematics; accurate belief about children's abilities to learn mathematics; and knowledge about mathematics teaching methods that are appropriate with children's development.

Furthermore, the result of this research gives a contribution to the stakeholders who are willing to increase the quality of ECE teachers through the professional development program. In this case, the researcher suggests that professional development programs should be focused on improving teachers' skills in childhood developmental stages. As an example, stakeholders can apply the Cognitively Guided Instruction (CGI) model, which is an approach in teaching mathematics created from the strategy of children's solving problems. CGI identifies individual strategies to help teachers understand students' thinking so that teachers can direct students to mathematical understanding. A CGI program is an effort for teachers to adjust themselves with their students' mathematical understanding. In terms of the implementation, teachers in the CGI program meet once a week in a workshop. During this period, they are faced with a research-based model of students' mathematics ways of thinking.

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