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Pierre Kieren's theory: the folding back process in mathematical problem solving

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ABSTRACT

The learning of mathematics generally undergoes a less effective and less appealing learning process, resulting in students' perceived lack of mastery of the material. Consequently, students' insufficient understanding of the concepts leads to a lack of folding back. In the process of understanding, it influences individual characteristics, where two characteristics are cognitive styles: field-dependent and field-independent. The researcher aims to understand how the folding back process occurs in students with field-dependent and field-independent cognitive styles when solving story problems. This research is a descriptive qualitative study, with 2 students selected from a total of 28 students in class VII-A as subjects. The selected subjects have high mathematical abilities and are classified into the categories of field-dependent and field-independent cognitive styles. Data collection involves comprehension tests, group embedded figure test (GEFT), and interviews. Data analysis consists of stages such as data reduction, data presentation, and verification. Each subject is interviewed to verify their process of solving the given problems. The results of the research conclude that students with the field-independent cognitive style category have a better understanding of the material, concepts, and problem-solving compared to students in the field-dependent category.

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1. INTRODUCTION

Education is one of the tools to improve the quality of a nation. Education can be pursued, among other ways, in schools, and one of the mandatory subjects to be understood in school is mathematics [1]. Mathematics is a familiar subject to students, as it is taught from elementary to university levels, given its significant role in education and everyday life [2]. Therefore, in mathematics education, it is crucial for students to understand mathematical concepts [3]. Through understanding, students can grasp the use of concepts, solve problems, describe, and predict events [4]. Understanding mathematical material can help organize and facilitate learning, indicating the interconnectedness within mathematical content [5]. However, the learning process often encounters ineffective methods, resulting in suboptimal learning experiences [6].

The suboptimal learning process is particularly evident when students are faced with story problems [7]. Students often need time to comprehend the problems due to difficulties in understanding concepts, deciphering the question's intent, translating sentence structures, or being less precise in calculations [8], [9]. Nevertheless, students can engage in "folding back" to comprehend and solve such questions. Folding back is a process of returning to the initial understanding when encountering difficulties in problem-solving [10]. The essence of folding back is to achieve a broad mathematical understanding. Piere Kieren [11]–[15] explains a theory of understanding with eight layers, including primitive knowing (basic knowledge of students), image-making (creating mental images), image-having (understanding created images), property noticing (reviewing images and considering instructions), formalizing (determining or formalizing a concept), observing (combining formalizing layers to find a solution), structuring (the process of structuring a solution), and inventing (discovery or final outcome). In Nabavi and Fossen [16], it is suggested that understanding is a multi-layered, non-linear, and limitless developmental process, illustrated as layers of an onion, each having its uniqueness with the presence of folding back. In the process of folding back, there are strategies employed to expand and develop existing understanding and to solve newly encountered problems [17]. It is mentioned that the growth of understanding in this theory is a dynamic and active process involving development and action, with constant movement among various levels of thinking without the involvement of a linear system. Therefore, the developmental process of students' thinking, which can involve folding back, influences the students' understanding. The understanding process of students significantly affects the effectiveness of learning, influencing information reception and habits. This is related to the learning environment, which is closely associated with cognitive styles [18]. Cognitive styles are individual characteristics of learners in responding to all received information [19].

According to Saha and Sharma [20] cognitive styles involve characteristics such as sensing, remembering, problem-solving, and decision-making in each individual. Addinna *et al.* [21] explain that the characteristic of field-dependent cognitive style tends to be group-oriented and dependent on group decisions, while field-independent cognitive style represents individuals who are more independent and not reliant on a group. Therefore, the researcher's focus is to understand the characteristics of field-dependent and field-independent cognitive styles in the high category that aligns with the conditions encountered by the author in the field. Hence, the researcher decided to explore how students engage in folding back while solving story problems, considering their cognitive styles in the high category.

Many researchers have examined the process of mathematical understanding based on the layers of folding back referring to the Pirie-Kieren theory, including Patmaniar *et al.* [13] who investigated students' folding back when solving arithmetic sequences problems and Irvine [22] in dynamic model. Ma'rifatin *et al.* [23] explored students' understanding in solving geometric problem, Mustikaningtyas and Susiswo [24] who examined folding back of a Kepanjen Islamic senior high school student in solving the function problems and their intervention, and Utomo *et al.* [25] the growth of understanding of high-achieving students in solving linear programming problems, and Chuene *et al.* [26] analyzed supports learners' folding back for growth in understanding geometry. However, no one has yet analyzed the process of mathematical understanding based on the layers of folding back in terms of cognitive styles in the topics of lines and angles. According to the background of this research, the author aims to describe in detail how layers of students' understanding with high comprehension abilities based on Piere Kieren's cognitive theory in solving story problems are examined from a cognitive style perspective.

2. METHOD

In this research, the author employs a descriptive research design with a qualitative approach, aiming to attempt to depict and interpret in accordance with field conditions supported by a methodology that investigates a phenomenon or social problem [27] to find results in line with the researcher's objective, which is to describe the folding back process of students in solving line and angle story problems based on cognitive style. The research process was carried out at State Junior High School 1 Kroya. During the research implementation process, the researcher utilized mathematical comprehension instruments, including question instruments and interview instruments. The instruments used in this study had previously undergone validation stages by experts in Table 1.

Based on the four validators in Table 1, the researcher's questions have been successfully validated, making them suitable for use in the research process. The indicators used in this study involve mathematical problem-solving, where the presentation of a concept and the planning of strategies/methods to solve problems, and the transformation of real-world problems into mathematical forms or vice versa, along with the use of symbols in performing arithmetic operations on the problems. The researcher selected the subjects from class VII-A, consisting of 28 students, by choosing students with high mathematical abilities in mathematics learning and students with cognitive styles categorized as field-dependent and field-independent. To determine these categories, the researcher used the group embedded figure test (GEFT) test previously employed by Witkin *et al.* [28] assisted by interviews. The GEFT test itself consists of three sections, where part 1 comprises 7 practice questions, while parts 2 and 3 each consist of 9 more complex questions than the practice ones. Each section of the test is given a relatively short time (around 15 minutes). To determine the expected results, the researcher employed data analysis techniques, following the approach of Miles and Huberman, which includes data reduction activities, data presentation, and conclusion drawing/verification [29], [30]. The process of data reduction involves sorting, organizing, and filtering data into smaller and grouped parts. This may involve creating abstractions, selecting the most relevant data, or narrowing the focus of analysis. After the data has been reduced, the next step is to organize the data in a format that facilitates analysis. This can be done by creating tables, diagrams, or matrices to visualize the relationships between various data elements [31]. The researcher then draws conclusions regarding the folding back process in mathematical problem solving. Subsequently, the researcher validated the research results using triangulation.

Table 1. Validation results by experts

Validator	Institutions	Information
2 mathematics education lecturers	Universitas Swadaya Gunung Jati	Valid
2 subject teachers	State Junior High School 1 Kroya	Valid

3. RESULTS AND DISCUSSION

According to the GEFT test, there are 8 students in the field-independent category and 20 students in the field-dependent category. Table 2 are the results of the categorization of subjects into cognitive styles. Based on the Table 2, it is stated that there are 8 students classified as field-independent and 20 students classified as field-dependent. After determining the students' cognitive style categories, the researcher conducted a mathematical test to identify subjects with high mathematical understanding. The researcher used the assessment criteria [32] as a reference for scoring the mathematical test, with scores categorized as low <3, medium $3\geq8$, and high >8. The two questions tested had scores of 5 each. Based on the scoring, students were categorized as follows: 6 students with a high category, 19 students with a medium category, and 3 students with a low category. To select research subjects, the researcher considered the results of tasks and other factors. Thus, two subjects were identified: S1, a subject with high mathematical ability and a field-dependent cognitive style (S1-FDH), and S7, a subject with high mathematical ability and a field-independent cognitive style (S7-FIH). In the process of data collection, aiming to generate research results in accordance with the established indicators, both subjects with field-dependent cognitive styles (S1-FDH) and field-independent cognitive styles (S7-FIH) successfully surpassed layers of mathematical understanding. In the case of subject S1-FDH, they successfully navigated through several layers, starting from image making, image having, property noticing, formalizing, and observing. However, upon entering the structuring layer, the student encountered difficulties when solving a problem involving finding <HGB. The subject underwent a folding back process to the formalizing layer to reflect on how to relate a concept to a problem solution, as the interview with researcher (R) below:

R : "Could you illustrate how you worked on the problem you solved?" Image making

FDH : "Sure, I can. Point A, Rian stands there and walks eastward to point B. Rian then turns 33°, there's point G, and point H towards the east forming <GHL 147°. In the question, there's an additional instruction about line X intersecting at point G."

R : "Do you know what was asked in the problem you worked on?"

FDH : "Yes, I knew. I was asked to find the value of <HGB."

R : "What steps did you use to find <HGB?"

FDH : "First, I drew the diagram, then looked for what was asked. After that, I marked point X. There are <GBA and <GBX, both measuring 33°, and there are also supplementary angles, <HGX and <IHG, so their sum is 180°. However, <HGX is not known, so it needs to be calculated first. Adding it to <IHG, <IHG being 147°, we subtract that from 180°, which gives 33°. So, in my opinion, the result for <HGX is 33°."

R : "Was there any other process that needed to be done after that?"

FDH : "So, finding <HGB, how is it? I'm confused."

R : "Try to understand the solution process again that you have done in your own answer, and recall the material taught by your teacher."

FDH : "Just add it, right? So that <HGB can be obtained, with <HGX and <BGX, let's just add them."

R : "Try to explain!"

FDH "Yes, so the <BGX is 33°, then the <HGX is 33° so that remains Just add the numbers and get the result <HGB is 66°."

Table 2. Category of cognitive style scores

Category	Score	Students
Field-dependent	0-9	8
Field-independent	10-18	20

Based on the interview results, it can be observed that the high ability field-dependent (FDH) student was able to mention information regarding the presented question by stating what was known and asked, which was for the subject to find <HGB. The subject explained the initial understanding found in the instructions of the question. The subject elucidated the initial knowledge of the question by mentioning that the question involved <GBA and <GBX having the same angle, and <HGX and <IHG being supplementary angles with a value of 180°, facilitating the problem-solving process because the value of <IHG was known to be 147°, so the subject calculated 180°-147° to obtain a result of 33°. The subject was aware of the next step, which was to find <HGB, but encountered difficulties in the process. Subsequently, the researcher provided the subject with an opportunity to reconsider the question and gave guidance to identify the subject's points of difficulty. The subject experienced a process of folding back where they revisited the layer of property noticing, illustrating it again with diagrams showing the value of each angle. The subject explained that <BGX had a value of 33°, and <HGX had a value of 33° as well. The subject added these two angles together, BGX+HGX=66, thus resulting in <HGB being 66°. Figure 1 is the results of the solution and the folding back process.

Based on Figure 1, it is stated that the subjects successfully completed the problem-solving process. However, the author cannot yet conclude whether the subjects engaged in the folding back process. Therefore, the researcher illustrates the thought process of the students in solving the problems as in Figure 2.

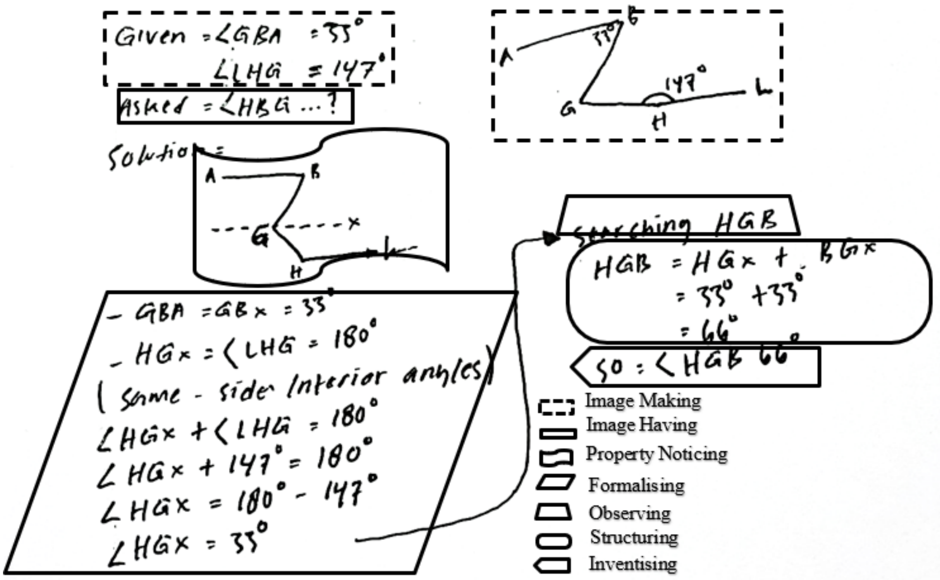

Figure 1. The result from high ability field-dependent student

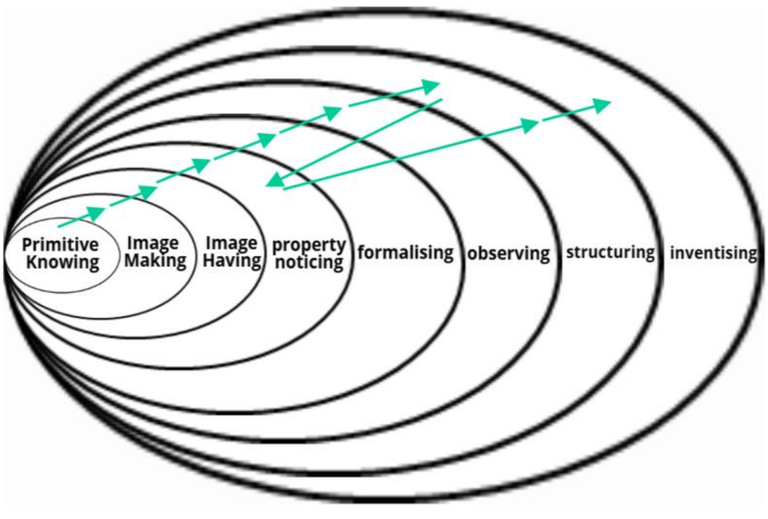

Figure 2. Illustration of understanding layers from high-ability field-dependent (FDH) student

Based on Figure 2, it is stated that subject S1-FDH underwent the folding back process once at the structuring layer, returning to the formalizing layer to obtain the solution for the problem, which is to determine the result of <HGB. Next, subject S7 with high-ability field-independent (S7-FIH) students successfully passed through several layers starting from image making, image having, property noticing, until finally reaching the formalizing layer. The subject faced challenges because they could not create or apply a mathematical concept based on the properties according to the problem. Therefore, the subject returned to the property noticing layer to understand the problem again, as reinforced in the following statement:

FIH : "I was asked to find <HGB. Now, in the diagram, create a point x that intersects point g. If you can understand the diagram, it is just a bit confusing when determining what to do first."
R : "Understand the solution you previously did."
FIH : "Yes, because there is an explanation that <GBA is 33, and if <LHG is 147, it becomes easier."

When the subject tried to understand again and successfully grasped some concepts related to the solution, they faced another challenge. The subject still lacked analysis or attempts to find the correct solution. In the interview process, the subject explained that they revisited the image-making layer, as follows:

R : "It is easier if you don't understand it yet. Try to understand the diagram that already has a point x intersecting g, understand it well."
FIH : "There is an opposite angle, such as angles <GBA, <GBX, and then look at the diagram again. There is also an angle on one side, such as <HGX and <LHG, so the value is 180°. So, the one who doesn't know the value of <HGX, if the first one to find is <HGX, then just add it to find <HGX, meaning <HGX+<LHG=180. The value of <LHG is already 147, so just put it in, so move 180-147=33."
R : "Was there any other process that needed to be done after that?"
FIH : "Yes, there is, just finding the value of <HGB."
R : "Can you explain the steps to find angle HGB?"
FIH : "So, to make it easier, let's redraw the diagram, because HGX=33°, and BGX is opposite to GBA, so BGX is also 33°. Then, just by looking at the diagram, we already know the result of HGB, we just need to add them up, so BGX+HGX=66."

Based on the interview results above, it can be understood that the subject FIH was able to mention information about the presented problem by stating what was known and what was asked, which was to find <HGB. The subject explained the initial understanding found in the instructions of the problem. The subject elaborated on the initial knowledge and how to obtain the initial step in the problem by mentioning that the problem had additional instructions where X intersects point G. In the problem, the student also re-illustrated the diagram with additional instructions to facilitate the problem-solving process. In the resolution, the student faced difficulties in understanding the problem, thus requiring the student to undergo a folding back process from the formalizing layer to the image making layer to facilitate the student's understanding process. Through the folding back process, the student knew the next step to take, which was to determine <HGX.

Next, the subject FIH encountered difficulties again regarding the process of finding <HGX, thus experiencing the second folding back to the property noticing layer to verify that the steps taken by the subject were correct. With the help of the diagram created by the subject, it was shown that the subject successfully understood the working steps by knowing that <GBA and <GBX were opposite angles, thus having the same value of 33, and that LHG and HGX were supplementary angles where if the supplementary angle had a value of 180, the subject found the value of <HGX by adding <HGX+<LHG=180°, entering the value of <LHG 147° facilitated the working process, so the subject added <HGX+147°=180°, with 147° moving to the right side, thus HGX=180°-147°=33°. The subject FIH knew that the value of HGX was 33°. Then, the subject FIH knew the next step to take by finding <HGB, the subject FIH found <HGB by re-illustrating the diagram with the values of each known angle, which made it easier for the subject to find <HGB. The subject explained that <HGB could be answered directly just by looking at the diagram, adding <BGX+<HGX because each angle was already known to be of a certain value, <BGX+<HGX=33°+33°=66°, thus the subject FIH stated that <HGB had a value of 66°. After completing the problem, the subject was confident in their answer and did not encounter difficulties in solving the given problem. Therefore, the researcher illustrates the thought process of the students in solving the problems as in Figure 3.

The Figure 4 are the results of the subject's solution and the folding back process that has been conducted. Based on Figure 4, it is stated that the subjects successfully completed the problem-solving process. However, the author cannot yet conclude whether the subjects engaged in the folding back process. From the two subjects above, it is evident that they have different folding back processes. Therefore, the author conducted data triangulation from both subjects as shown in Table 3.

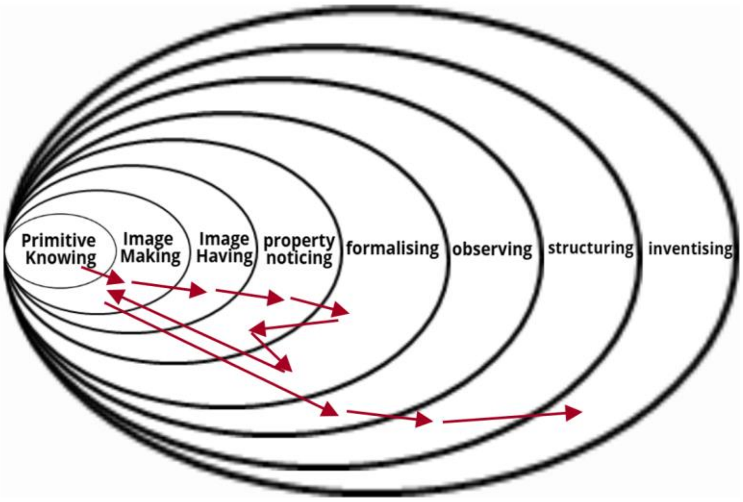

Figure 3. Illustration of understanding layers from high ability field-independent students

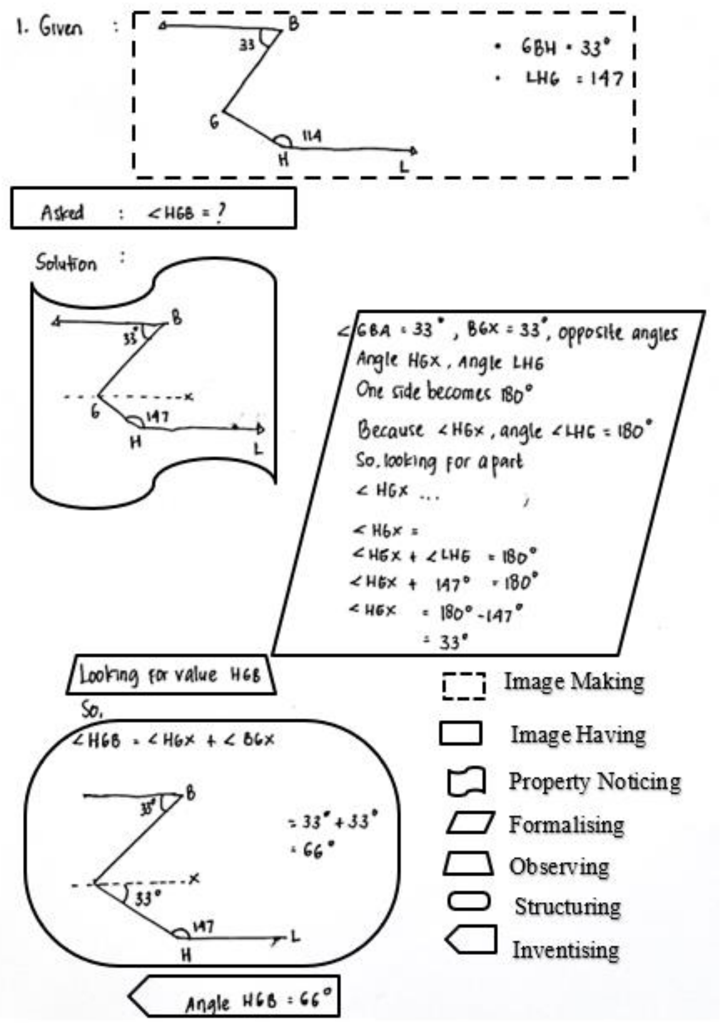

Figure 4. Work students result (FIH)

Table 3. Presentation of conclusion: folding back process of students with cognitive style (field-dependent and field-independent)

No	Layer folding back	Field-dependent	Field-independent
1	Primitive knowing	Having an initial understanding by being familiar with the material to be tested.	Having an initial understanding by being acquainted with the material to be tested
	Conclusion	Subjects S1 and S7 have an initial understanding related to the material of lines and angles.	
2	Image making	Capable of creating an overview as a general stage of problem-solving	Capable of creating an overview as a general stage of problem-solving
	Conclusion	Subjects S1 and S7 are able to generate a general overview by understanding the issues in the problem	
3	Image having	Able to identify the problems in the questions without working on examples	Able to understand the issues in the questions without working on examples
	Conclusion	Subjects S1 and S7 are able to identify the issues in the questions without solving them in detail and working on examples	
4	Property noticing	Successfully connecting with the overview of the problem without providing detailed explanations	Successfully connecting with the overview of the problem without explaining in detail
	Conclusion	Subjects S1 and S3 are capable of connecting the general overview they possess with the concept of planning in problem-solving	
5	Formalising	Capable of applying the known problem-solving process at the previous level	Capable of applying the known problem-solving process with improvements, resulting in two-fold feedback on primitive knowing and image making
	Conclusion	Subject S1 is capable of applying problem-solving without correction, while subject S7 undergoes a two-fold folding back process by returning to the layers of primitive knowing and image making, making the subject considered proficient after completing this folding back process.	
6	Observing	Capable of observing the solution with improvements, leading to a folding back to the level of property noticing	Capable of making observations from the previous problem-solving
	Conclusion	Subject S1 undergoes a folding back process and returns to the layer of primitive knowing, making the subject considered proficient after completing this folding back process. Subject S7 is capable of observing the solution from the problem-solving at the previous level	
7	Structuring	Compiling the solution to the problems related to the material on lines and angles based on the problem-solving process at the previous level.	Compiling the solution to problems related to the material on lines and angles based on the problem-solving process at the previous level
	Conclusion	Subjects S1 and S7 are capable of solving from the previous stage up to the point of determining the results of the problem-solving process	
8	Inventising	Unable to generate new questions based on the material learned but can fully complete given problems	Unable to create new questions based on the material learned, but can successfully complete the given problems
	Conclusion	Subjects S1 and S7 are able to solve problems but are unable to generate new statements from the problems they have worked on	
9	Folding back	Returning to the deepest layer of the previous level of understanding to solve a problem without deviating from the topic, done once.	The occurrence of the process of returning to the deepest layer of the previous level of understanding to solve a problem without deviating from the topic, done twice
		- Subject S1, with a field-dependent cognitive style, can solve problems through a single folding back process from the observing layer to the formalizing layer. - Subject s7, with a field-independent cognitive style, can solve problems with two folding back processes from the formalizing layer to the property noticing layer. However, the subject is still unable to proceed to the next level, leading to a folding back to the image making layer until they can advance to the next level	

Based on the Table 3, it is stated that subject S7-FIH underwent the folding back process twice, returning from the formalizing layer to the property noticing layer. However, when in the property noticing layer, the subject returned again to the formalizing layer, experiencing challenges that required searching for and understanding a concept from the problem. This led to a return to the image-making layer. Then, it can be illustrated using the layers of understanding in Pierre Kieren's theory as shown in Figure 5.

Based on Figure 5, there is a difference in the folding back process between the two subjects. In the study Galiakberova and Galyamova [33], it is stated that students with a field-dependent cognitive style generally can only create images but cannot provide explanations for the created images. Thus, for subject S1-FDH, there is a single folding back process from the structuring layer where the subject is able to observe the problem-solving process. This leads to a folding back to the property noticing layer, which involves re-examining the solution process to connect a problem to proceed to the next layer, allowing the subject to fully complete the problem. The process experienced by S1-FDH follows the sequence of folding back: PK-IM-IH-PN-F-O-S-PN-S-I.

For subject S7-FIH, in line with the research by [34], students with a field-independent cognitive style tend to undergo a more specific understanding process due to a curiosity to solve problems. In FIH, there are two folding back processes from the formalizing layer. The subject is unable to apply the known problem-solving process, leading to a folding back to the property noticing layer. The subject can connect concepts without detailed explanations but cannot proceed to the next layer. As a result, there is a folding back from the formalizing layer to the image-making layer, involving a re-examination of the general overview of the problem to proceed to the next layer. The process experienced by S7-FIH follows the sequence of folding back: PK-IM-IH-PN-F-PN-F-IM-F-O-S-I.

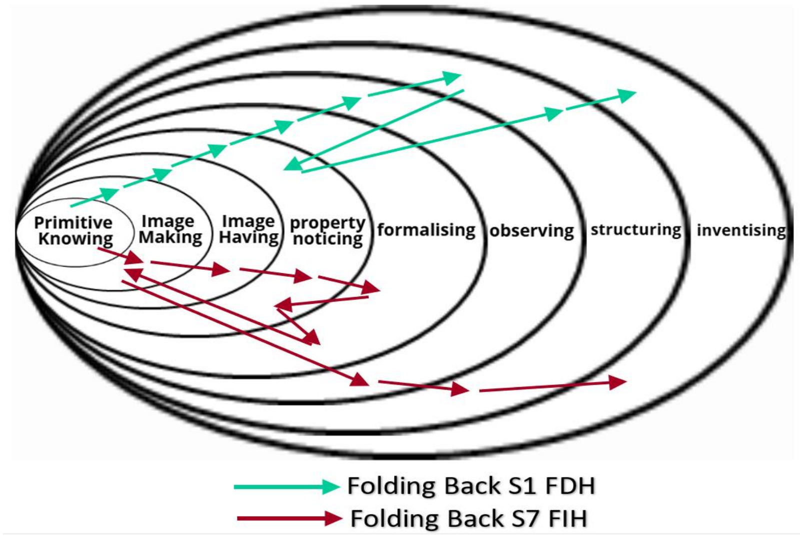

Figure 5. Folding back of students with cognitive style

According to Susiswo *et al.* [35], the more frequent the occurrence of folding back, the deeper the student's understanding of a mathematical concept they have learned. To enhance understanding of learned mathematics, it is recommended to use learning assistance to increase student concentration. Learning assistance [36] plays a crucial role in the teaching and learning process. Scaffolding is essential in addressing mathematics learning challenges as it encourages active student participation and enhances understanding of the material being discussed. According to Puntambekar [37], scaffolding has three levels: basic environmental provisions, direct interactions between teachers and students through explaining, reviewing, and restructuring, and emphasis on conceptual thinking. However, in this study, each student has unique thinking processes, categorized earlier as either field-dependent or field-independent cognitive styles. The author suggests adding or combining students' mathematical understanding processes with learning assistance. It is designed to help students understand mathematical concepts gradually and deeply. In the context of mathematics learning, this approach has several significant benefits. Firstly, by providing a supportive learning environment, scaffolding helps reduce anxiety and boost students' confidence in solving complex mathematical problems [38]. Secondly, through direct interactions between teachers and students, both individually and in small groups, students receive personalized and specific guidance according to their needs [39]. This helps direct students' thinking towards the correct problem-solving processes and reinforces their understanding of difficult mathematical concepts. Additionally, by emphasizing conceptual thinking, scaffolding encourages students to grasp the fundamental ideas behind mathematical concepts, rather than just memorizing formulas or procedures [40], [41]. For field-dependent students, learning assistance at the direct interactions level, where students interact directly with teachers and peers through explanation, review, and restructuring, could be beneficial. For field-independent students, learning assistance at the environmental provisions level, involving the provision of a supportive learning environment, may be effective.

4. CONCLUSION

Based on the research results, the process of students' folding back in solving story problems, examined from the cognitive styles of field-dependent and field-independent, indicates that the more frequent the occurrence of the folding back process, the deeper the students' understanding of mathematics. In this study, students with the field-independent cognitive style more frequently experience folding back twice, with the sequence of folding back processes being PK-IM-IH-PN-F-PN-F-IM-F-O-S-I. Meanwhile, field-dependent students experience folding back only once, with the process sequence being PK-IM-IH-PN-F-O-S-PN-S-I. Thus, students with a field-independent cognitive style tend to have a better understanding of the material, concepts, and problem-solving compared to field-dependent students. As a suggestion for future research, the researcher recommends adding scaffolding or learning assistance using instructional media. This is expected to enhance the process of students' mathematical understanding in a more critical manner.

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