

Students' Semantic-Proof Production in Proving a Mathematical Proposition

Syamsuri¹, Purwanto², Subanji³, Santi Irawati⁴

¹Department of Mathematics Education, Universitas Sultan Ageng Tirtayasa, Indonesia

^{2,3,4}Department of Mathematics, Universitas Negeri Malang, Indonesia

Article Info

Article history:

Received Sep 2, 2017

Revised Apr 27, 2018

Accepted May 11, 2018

Keywords:

APOS theory

False-semantic proof

Mathematical proposition

Proving

Semantic-proof

ABSTRACT

Proving a proposition is emphasized in undergraduate mathematics learning. There are three strategies in proving or proof-production, i.e.: procedural-proof, syntactic-proof, and semantic-proof production. Students' difficulties in proving can occur in constructing a proof. In this article, we focused on students' thinking when proving using semantic-proof production. This research is qualitative research that conducted on students majored in mathematics education in public university in Banten province, Indonesia. Data was obtained through asking students to solve proving-task using think-aloud and then following by interview based task. Results show that characterization of students' thinking using semantic-proof production can be classified into three categories, i.e.: (1) false-semantic, (2) proof-semantic for clarification of proposition, (3) proof-semantic for remembering concept. Both category (1) and (2) occurred before students proven formally in Representation System Proof (RSP). Nevertheless, category (3) occurred when students have proven the task in RSP then step out from RSP while proving. Based on the results, some suitable learning activities should be designed to support the construction of these mental categories.

Copyright © 2018 Institute of Advanced Engineering and Science.
All rights reserved.

Corresponding Author:

Syamsuri,

Department of Mathematics Education,

Universitas Sultan Ageng Tirtayasa,

Jalan Raya Jakarta KM. 4 Pakupatan Kota Serang, Provinsi Banten, Indonesia.

Email: syamsuri@untirta.ac.id

1. INTRODUCTION

Mathematical proof is important component in mathematics learning. As well as knowledge, mathematics develops through mathematical proof. Besides, a mathematical proposition can accepted as knowledge if it has a correct proof. Therefore, mathematical proof is like a pillar in mathematics building [1].

The purpose of activity in constructing a proof is to prove and to explain a proposition [2]. As well as Hanna suggested that proof as bearer a knowledge in context of mathematics learning [3]. Role of mathematical proof is bearer of mathematical knowledge, because a proof contains a new knowledge that constructed using knowledge before. Moreover, mathematical proof can generate problem-solving process to think on constructing a mathematical knowledge [4].

Proving a proposition emphasize in undergraduate mathematics learning. Therefore, undergraduate mathematics learning is different from school mathematics learning. In undergraduate learning put emphasis on logics and rigorous mathematical thinking. For instance, at mathematical school, vector as mathematical object that have magnitude and direction, also vector can be displayed as arrow or others symbol. Nevertheless, at undergraduate learning, vector was defined axiomatically and constructed through formal deductive. A proposition is valid if there is proof that showed from axioms and some definitions using logics rule. Proving process use formal approach and can behave a condition and appropriate to proving context. Therefore, mathematics learning in university involves mental confusion as connection among perception

and action, and then reorganize formally deductive, so that can build a new experience through formal situation [5].

In students' thinking process in constructing proof, Pinto and Tall used term 'natural' to describe the process of extract-meaning and term 'formal' to describe process of giving-meaning that work formally [6]. In addition, Weber added idea of 'procedural' learning for students who are just trying to cope with formal definitions and proving with rote learning [7]. Alcock and Weber customized student response to 'semantic' and 'syntactic', based on language which refers to the meaning of language (semantic contents) and grammar (syntax) [8]. Alcock and Weber described the syntactic approach as one strategy in proving that involved mathematical definition formally, and semantic approach as a strategy in utilizing intuitive understanding of a concept. It is actually in line with the term *extract-meaning* and *giving meaning* in the category of 'formal' and 'natural'.

In this regard, according to [4] stated that there are three strategies are usually done to prove. A first strategy is strategy of procedural proof production. In a procedural strategy of proof production, students prove by looking for proof of similar examples. Furthermore, students modify it according to the statement to be proved. Topics that are usually done with this strategy is the limit of a sequence and limit of function. A second strategy is syntactic proof production. Syntactic proof production strategy is a strategy that began with some definitions and assumptions are appropriate to the problem, and then draw conclusions based on the definitions and assumptions by utilizing existing theorems and rules of logic. A third strategy is the semantic proof production. Semantic proof production strategy is the strategy of mathematical proofs that use different representations of mathematical concepts informally to guide writing of strict and formal proof. The illustrations are, i.e.: graphs, drawings, or examples of cases. With hint of illustrations, the idea of proving is expected to appear. Alcock and Inglis [9] defined proof production using Representations System of Proof (RSP). RSP is a series of statement set based on logics and deductive to show that the proposition is true. According that, they stated that if all step to produce statements in proving on RSP, so then it is called syntactic-proof production. Moreover, it is called semantic-proof production when step out from RSP while proving.

Many studies have revealed that students have difficulties in constructing a mathematical proof [10-14]. The difficulties in proving can occurred in constructing a proof, so then made an incorrect proof. One strategy to refine a constructing a proof is how to encourage students to make sense through example, drawing a picture, sketch or other informal mathematical objects. We was suggested that semantic strategy can help students to aware about their error in constructing a proof [15]. This strategy encourages students to do reflective thinking so that they can be aware of their fault when constructing a proof.

In revealing a thinking process, this study is based APOS theory [16]. APOS theory proposes that an individual has to have appropriate mental structures to make sense of a given mathematical concept. The mental structures refer to actions, processes, objects and schema which required to learn the concept. Research based on this theory requires that for a given concept the likely mental structures need to be detected, and then suitable learning activities should be designed to support the construction of these mental structures. Base on explain above, thus article will describe characterization of students' thinking using semantic-proof production in proving a mathematical proposition.

2. RESEARCH METHOD

2.1. Participants

This research is descriptive exploratory study to reveal characteristics of semantic proof production in proving. Therefore, research approach used is qualitative. The research conducted on students majored in mathematics education in public university in Banten province, Indonesia. The consideration of that was students were able to think a formal proof in mathematics. Students who constructed semantic proof production was selected as research subject.

2.2. Instrument

The main instrument in qualitative research was researcher itself. The support instruments are proving-task and interview guides. These instruments were evaluated and validated from two lecturers in order to guarantee quality of instruments. The interview is open and it's needed to reveal students' response about error in constructing a proof. Procedure to obtain data are 1) subject is given the task-proving and asked him/her to accomplish the task by think-aloud. And then 2) subject is interviewed base on the-task. Therefore, scratch of proving-task and transcript of interview is obtained. The proving-task is in the following.

Prove: For any positive integers m & n , if m^2 and n^2 are divisible by 3, then $m+n$ is divisible by 3.

We used this task because some methods can use for solving, i.e.: direct proof, contradiction, and contra-positive. Besides, we would like to test students' comprehension about mathematical induction method, because some students have an opinion that using mathematical induction to prove a number which "divisible by 3". This research is descriptive exploratory study to reveal characteristics of semantic proof production in proving. Therefore, research approach used is qualitative.

2.3. Data Analyse

Data obtained are recording of think-aloud, scratch of proving-task-sheet, and recording of interview. These data was collected to describe students' thinking process in constructing a semantic proof production. Therefore, reliability of this research is source of data.

To analyse students' thinking process, we use APOS theory. In APOS theory, the main mental mechanisms for building the mental structures of action, process, object, and schema are called *interiorization* and encapsulation [17]. The mental structures of action, process, object, and schema constitute the acronym APOS. APOS theory postulates that a mathematical concept develops as one tries to transform existing physical or mental objects. The descriptions of action, process, object and schema in this research are given below:

- a. Action: A transformation is first conceived as an action, when it is a reaction to stimuli which an individual perceives as external. Action's indicator is student has given some correct-example for the proposition.
- b. Process: As an individual repeats and reflects on an action, it may be interiorized into a mental process. A process is a mental structure that performs the same operation as the action, but wholly in the mind of the individual. Process' indicator is students could create mathematical equation model into variable form.
- c. Object: If one becomes aware of a process as a totality, realizes that transformations can act on that totality and can actually construct such transformations (explicitly or in one's imagination), then we say the individual has encapsulated the process into a cognitive object. For example, students could create other representation from mathematical equation model in order to connect to other information in the proposition.
- d. Schema: A mathematical topic often involves many actions, processes, and objects that need to be organized and linked into a coherent framework, called a schema. It is coherent in that it provides an individual with a way of deciding, when presented with a particular mathematical situation, whether the schema applies. For example, there are good connection among integer concept, divisibility concept, logic of implication in the proposition.

3. RESULTS AND ANALYSIS

Of the 25 students who were given the proving-task. There were 12 students who constructed proof using semantic-proof production. The remaining 13 students are constructed using syntactic proof production. The semantic-proof production is occurred before students prove formally in Representation System Proof (RSP) or when students prove in RSP. Of the 12 students who constructed proof using semantic-proof production, their thinking process can be classified into 3 categories, i.e.: false-semantic, proof-semantic for clarification of proposition, and proof-semantic for remembering concept. False-semantic and semantic-proof for clarification of proposition occurred before students prove formally in Representation System Proof (RSP). And semantic-proof for remembering concept occurred when students prove in RSP.

3.1. False-semantic Proof Production

There are 4 students who constructed false-semantic proof. Based on our observation, founded that some students tried to make sense of proposition by incorrect illustration. Proving using incorrect illustrations called "false-semantic"[18]. Figure 1 shows an example of false-semantic proof production. For instance, S1 given illustration for the proving-task by choosing $m=1$ and $n=1$, so that $m^2=1^2=1$ is not divisible by 3. So then, S1 concluded that the proving-task is unproved. Other student, namely S2, she given an example in form even number, $m=2$ and $n=2$ are not divisible by 3, whereas the example is given by S1/S2 are incorrect.

Handwritten proof (left):

1. misalkan $m=1$ dan $n=1$, maka $m^2 = (1)^2 = 1$ tidak habis dibagi 3
 $n^2 = (1)^2 = 1$ tidak habis dibagi 3
 jadi $m+n$ habis dibagi 3 itu tidak terbukti
 $m+n = 1+1 = 2$ tidak habis dibagi 3
 tidak terbukti
 $m=2$ dan $n=3$, maka $m^2 = (2)^2 = 4$ tidak habis dibagi 3
 $n^2 = (3)^2 = 9$ habis dibagi 3
 $m+n = 2+3 = 5$ tidak habis dibagi 3.

Translated English (right, in a dashed box):

Translate to English:
 Suppose $m=1$ and $n=1$, then
 $m^2=(1)^2=1$ is not divisible by 3
 $n^2=(1)^2=1$ is not divisible by 3
 The proposition is not proved.
 $m+n=1+1=2$ is not divisible by 3
 Thus, $m+n$ is not divisible by 3
 $m=2$ and $n=3$, then $m^2=(2)^2=4$ is not divisible by 3
 $n^2=(3)^2=9$ is divisible by 3
 $m+n=2+3=5$ is not divisible by 3

1a. A false-semantic proof is written by S1

Handwritten proof (left):

misal m dan n bilangan ganjil
 diambil contoh: $m=3 \rightarrow m^2=9$
 $n=3 \rightarrow n^2=9$ } habis dibagi 3
 $m+n$ habis dibagi 3
 $3+3$ habis dibagi 3
 misal m dan n bilangan genap
 diambil contoh: $m=2$
 $n=2$ } tidak habis dibagi 3

Translated English (right, in a dashed box):

Translate to English:
 Suppose m and n are an odd number,
 Let $m=3 \rightarrow m^2=9$ is divisible by 3
 $n=3 \rightarrow n^2=9$ is divisible by 3
 $m+n$ is divisible by 3
 Suppose m and n are an even number,
 Let $m=2 \rightarrow m$ is not divisible by 3
 $n=2 \rightarrow n$ is not divisible by 3

1b. A false-semantic proof is written by S2

Figure 1. Example of False-Semantic Proof Production

According to Balacheff [19], a proof was constructed by either S1 or S2 is *naïve-empirism*, but incorrect illustration. S1/S2 didn't have a good comprehension about the proposition. Understanding about content of proposition is important to do next proving step. Andrew stated two main components in evaluation students' proof, that are conceptual understanding and proof-structure [20]. If conceptual understanding was not appeared, so then students will be difficult to construct a proof. It is in line with Moore [10] who stated that students have difficult how to begin a proof.

Corresponding to scratch of student proof of S1/S2, we can see that both S1 and S2 have done 'Action' using illustration in order to understand the proving-task proposition. Yet S1/S2 was gave incorrect illustration. Corresponding to transcript of think-aloud, both S1 and S2 was thought "For any positive integers m & n , if m^2 and n^2 are divisible by 3, then ..." by only emphasized on "For any positive integers m & n ", neither S1 and S2 was thought on "if m^2 and n^2 are divisible by 3". Its showed that both S1 and S2 thought imperfect on proof-structure, so that they didn't use all information in the task-proving proposition. Therefore, they done imperfection "Action", so then they didn't appear *interiorization* "Action" into "Process".

Students' characteristics in false-semantic have no proof-structure, and also have a little conceptual understanding. They done imperfection "Action", so then they didn't perform *interiorization* "Action" into "Process". Therefore, the students need assistance to refine a proof-structure and conceptual understanding about proposition. One of method is learning using worked-example [7]. Worked-example is example how to proving a proposition, and involved arguments in every step. Worked-example can refine students' knowledge, 1) how to begin a proof, 2) how to understand about end of proof, 3) how to give argumentation for each step, and 4) how to select mathematical concept is needed. Point 1) and 2) related to refine proof-structure, in addition point 3) and 4) related to refine conceptual-understanding.

3.2. Semantic-proof for Clarification of Proposition

There are 5 students who constructed proof-semantic for clarification of proposition. Based on our observation, founded that some students tried to make sense of proposition by correct illustration. Figure 2 shows an example of semantic-proof for clarification of proposition. For instance, S3 given illustration for the proving-task by choosing $m=6$ and $n=3$, so that $m^2=6^2=36$ and $n^2=3^2=9$ are divisible by 3. So then, S3 concluded that the proving-task is proved. Other student, namely S4, she given an example in form even number, $m=6$ and $n=6$ are divisible by 3.

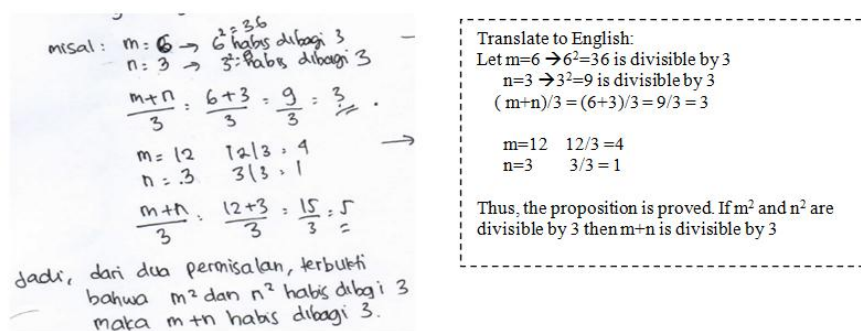


Figure 2. Example of semantic proof for clarification of proposition is written by s3

According to Balacheff [19], a proof was constructed by S3/S4 is *naïve empirism*. Both S3 and S4 only recalled an example of proposition. Students in this group could not use high order thinking for analysing in order to connect to other concepts. Conceptual understanding about content of proposition is partial, it's showed that proof-schema was not built. Neither S3 nor S4 have connected concept well. Besides, they didn't appear proof-structure, because they didn't definite a first step in proving. The first step is important for next step [10].

Corresponding to scratch of student proof of S3/S4, we can see that both S3 and S4 have done 'Action' well using illustration in order to understand the proving-task proposition. Both S3 and S4 was gave correct illustration. Corresponding to transcript of think-aloud, both S3 and S4 were thought "For any positive integers m & n , if m^2 and n^2 are divisible by 3, then ..." well. Both S3 and S4 used information in hypothesis of task-proving. Nevertheless, S3/S4 couldn't bring up a connection among divisibility concept of m and n , so then *interiorization* wasn't appeared. Therefore, they done imperfection "Action", so then they didn't appear *interiorization* "Action" into "Process".

Students' characteristics in this group have no proof-structure. Therefore, the students need assistance to refine a proof-structure. One method for refining proof-structure is learning using structural method. This method is adopted from Leron [21]. The method triggered by recent ideas from computer science, is intended to increase the comprehensibility of mathematical presentations while retaining their rigor. The basic idea underlying the structural method is to arrange the proof in *levels*, proceeding from the top down; the levels themselves consist of short autonomous "modules," each embodying one major idea of the proof. The top level gives in very general (but precise) terms the main line of the proof. The second level elaborates on the generalities of the top level, supplying proofs for unsubstantiated statements, details for general descriptions, specific constructions for objects whose existence has been merely asserted, and so on. If some such sub procedure is itself complicated, we may choose to give it in the second level only a "top-level description," pushing the details further down to lower levels.

3.3. Semantic-proof for Remembering of Concept

There are three students who constructed proof-semantic for clarification of concept. The students in proof-semantic for remembering of concept have proven the task in RSP, then step out from RSP while proving. The aim of this strategy is to remember or clarify concept by example. Figure 3 shows an example of semantic proof for remembering of concept. For instance, S5 gave illustration whether if $(m+n)(m-n)$ is divisible by 3 then $(m+n)$ is divisible by 3. S5 wrote $(5+4)(5-1)/3$ and clarified whether its statement correct or incorrect. But actually the case is $3|(5+4)(5-1)$. Another students, S6 gave illustration for getting meaning of $a \equiv b \pmod{m}$ by $a=2m+b$ and $4 \equiv 7 \pmod{3}$. While S7 gave illustration for getting meaning of $a|b$ by $b=am+k$.

According to Balacheff [19], a proof was constructed by them is *thought-experiment*. They used high order thinking for analysing in order to connect to other concepts. Conceptual understanding about content of proposition is integrated, it's showed that proof schema was built. Nevertheless, they needed some aid for reinforcement a claim using semantic strategy. They have a good comprehension about mathematical concept and ability to connect them. Besides, they appeared proof-structure, because they defined a first step in proving well.

Corresponding to transcript of think-aloud, S5 was thought "For any positive integers m & n , if m^2 and n^2 are divisible by 3, then ..." well. S5 used integrated information in hypothesis of task-proving. S5 done "Action" well, so then S5 appeared *interiorization* "Action" into "Process". And then, encapsulation

‘Process’ into ‘Object’ is successful, it showed by correct claim that m^2+n^2 is divisible by 3. Nevertheless, when S5 claimed that $m+n$ is divisible by 3, S5 only paid attention to $m+n$ without $m-n$, but actually the case is $m-n$ is divisible by 3 too. Besides, notation is written by S5 was incorrect, get accidentally exchanged among divided and divisor. It’s indicated that Object is not formed, so that Schema is uncompleted too.

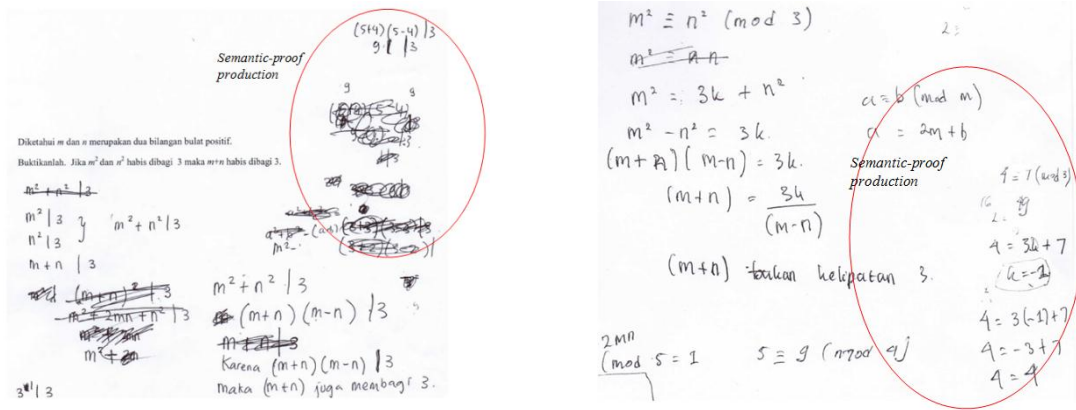


Figure 3. Example of semantic proof for remembering of concept is written by s5 (left) and s6 (right)

Corresponding to transcript of think-aloud, Both S6 and S7 were thought “For any positive integers m & n , if m^2 and n^2 are divisible by 3, then ...” well. Either S6 or S7 used integrated information in hypothesis of task-proving. They done “Action’ well, so then they appeared interiorization “Action’ into “Process”. And then, encapsulation ‘Process’ into ‘Object’ is unsuccessful, it showed by incorrect claim that $m+n$ is not divisible by 3. S6 argued that $m+n = \frac{3k}{m-n}$ is not divisible by 3. It’s indicated that encapsulation is unsuccessful. Meanwhile S7 only showed $m+n = \sqrt{3b+k_2} + \sqrt{3a+k_1}$. It’s indicated that S7 failed in encapsulation ‘Process’ into ‘Object’. It’s indicated that Object is not formed, so that Schema is uncompleted too.

Students’ characteristics in this group have a proof-structure, but a little of concept is forgotten. Therefore, the students need assistance to refine by remembering a needed concept. One method for refining conceptual-understanding is learning that emphasized meaning of concept, such as reading for meaning [22]. Accordingly, mathematical concept such as definition, notation or applied matter can be understood well.

4. CONCLUSION

Characterization of students’ thinking using semantic-proof production can be classified into 3 categories, that are false-semantic, proof-semantic for clarification of proposition, and proof-semantic for remembering concept. False-semantic and semantic-proof for clarification of proposition occurred before students prove formally in Representation System Proof (RSP), but semantic-proof for remembering concept occurred when students prove in RSP.

Students’ characteristics in false-semantic have no proof-structure, and also have a little conceptual understanding. They done imperfection “Action’, so then they didn’t perform interiorization “Action’ into “Process”. Therefore the students need assistance to refine a proof-structure and conceptual understanding about proposition. One of method is learning using worked-example.

Students’ characteristics in proof-semantic for clarification of proposition have no proof-structure. They could not use high order thinking for analysing in order to connect to other concepts. Conceptual understanding about content of proposition is partial, it’s showed that proof schema was not built. They done imperfection “Action’, so then they didn’t perform interiorization “Action’ into “Process”. Therefore, the students need assistance to refine a proof-structure. One method for refining proof-structure is learning using structural method.

Students in proof-semantic for remembering of concept have proven the task in RSP, then step out from RSP while proving. A proof was constructed by them is thought-experiment. They used high order thinking for analysing in order to connect to other concepts. They appeared interiorization “Action’ into “Process”, but Schema is not formed well. Students’ characteristics in these categories have conceptual-

understanding and proof-structure, but some concept was forgotten. Therefore, to the students need assistance to refine by remembering a needed concept. One method for refining conceptual-understanding is learning that emphasized meaning of concept, such as reading for meaning.

ACKNOWLEDGEMENTS

The first author gives appreciation and thanks to The Ministry of Research, Technology and Higher Education of Indonesia which provides the opportunity to continue his studies in mathematics education study program of Universitas Negeri Malang.

REFERENCES

- [1] D. Tall, O. Yevdokimov, B. Koichu, W. Whiteley, M. Kondrieteva, and Y. H. Cheng, "Cognitive Development of Proof," in *Proof and proving in mathematics education*, The 19th ICMI Study, G. Hanna and M. De Villiers, Eds. New York: *Springer*, 2012, pp. 13-49.
- [2] R. Hersh, "Proving Is Convincing and Explaining" *Educational Studies in Mathematics*, 1993, vol. 24, no. 4, pp. 389-399.
- [3] G. Hanna, "Proof, Explanation and Exploration: An Overview," *Educational Studies in Mathematics*, 2000, vol. 44, no. 1, pp. 5-23.
- [4] K. Weber, "Problem-solving, proving, and learning: The relationship between problem-solving processes and learning opportunities in the activity of proof construction," *Journal of Mathematical Behavior*, 2005, vol. 24, pp. 351-360.
- [5] D. Tall, "Perception, Operations and Proof in Undergraduate Mathematics," *Community for Undergraduate Mathematics Sciences Newsletter University of Auckland*, 2010, pp. 21-28.
- [6] M. M. F. Pinto and D. O. Tall, "Student constructions of formal theory: giving and extracting meaning," in *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education*, 1999, pp. 65-73.
- [7] K. Weber, "A framework for describing the processes that undergraduates use to construct proofs," in *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 2004, vol. 4, pp. 425-432.
- [8] K. Weber and L. Alcock, "Semantic and syntactic proof productions," *Educational Studies in Mathematics*, 2004, vol. 56, pp. 209-234.
- [9] L. Alcock and M. Inglis, "Doctoral students' use of examples in evaluating and proving conjectures," *Educational Studies in Mathematics*, 2008, vol. 69, pp. 111-129.
- [10] R. C. Moore, "Making the transition to formal proof*," *Educational Studies in Mathematics*, 1994, vol. 27, pp. 249-266.
- [11] L. Sowder and G. Harel, "Case studies of mathematics majors' proof understanding, production, and appreciation," *Canadian Journal of Science, Mathematics and Technology Education*, January 2015, vol. 3, pp. 251-267.
- [12] Syamsuri, Purwanto, Subanji, and S. Irawati, "Characterization of students formal-proof construction in mathematics learning," *Communications in Science and Technology*, 2016, vol. 1, no. 2, pp. 42-50.
- [13] Syamsuri, Purwanto, Subanji, and S. Irawati, "Using APOS Theory Framework: Why Did Students Unable to Construct a Formal Proof?," *International Journal on Emerging Mathematics Education*, 2017, vol. 1, no. 2, pp. 135-146.
- [14] Syamsuri and C. Santosa, "Karakteristik pemahaman mahasiswa dalam mengonstruksi bukti matematis," *Jurnal Review Pembelajaran Matematika*, 2017, vol. 2, no. 2, pp. 131-143.
- [15] Syamsuri, Purwanto, Subanji, and S. Irawati, "Facilitating Students From Inadequacy Concept in Constructing Proof to Formal Proof," in *3rd International Conference on Research, Implementation and Education of Mathematics and Science*, 2016, p. ME-34.
- [16] E. D. Dubinsky and M. A. McDonald, "APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research," in *The Teaching and Learning of Mathematics at University Level*, vol. 7, D. Holton, M. Artigue, U. Kirchgräber, J. Hillel, M. Niss, and A. Schoenfeld, Eds. Netherlands: Kluwer Academic Publishers, 2001, pp. 275-282.
- [17] K. Weller, J. Clark, and E. Dubinsky, "Student performance and attitudes in courses based on APOS Theory and the ACE Teaching Cycle," *Research in Collegiate Mathematics Education*, 2003, vol. 12, pp. 97-131.
- [18] Syamsuri, "Analysis of False-Semantic Proof Production in Undergraduate Mathematics Learning Based on APOS Theory," *IOSR Journal of Research & Method in Education*, 2016, vol. 6, no. 5, pp. 44-49.
- [19] N. Balacheff, "Aspects of Proof in Pupils' Practice os School Mathematics," *Mathematics, teachers and children*, 1988, pp. 216-235.
- [20] L. Andrew, "Creating a proof error evaluation tool for use in the grading of student-generated 'Proofs,'" *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 2009, vol. 19, no. 5, pp. 447-462.
- [21] U. Leron, "Structuring Mathematical Proofs," *The American Mathematical Monthly*, 1983, vol. 90, no. 3, pp. 174-185.
- [22] H. F. Silver, R. W. Strong, and M. J. Perini, *The Strategic Teacher*. Virginia: Association for Supervision and Curriculum Development (ASCD), 2007.

BIOGRAPHIES OF AUTHORS

Syamsuri, Dr., M.Si., S.Si. is currently an assistant professor in the Department of Mathematics Education at the Universitas Sultan Ageng Tirtayasa. He holds a S.Si. in Mathematics from Institut Pertanian Bogor (IPB) and a M.Si. in Applied Mathematics from IPB. He earned his Doctor degree in Mathematics Education from Universitas Negeri Malang (2017), with a concentration in thinking process reasearch on mathematics and its learning. His research spans the field of mathematics education and applied mathematics.



Purwanto, Prof., PhD., Drs. is currently a professor in the Department of Mathematics at the Universitas Negeri Malang. He holds a Drs. in Mathematics Education from IKIP Yogyakarta and a Ph.D in Mathematics from Curtin Univ. of Tech., Australia (1992). His research spans the field of combinatorics, graph theory and mathematics education.



Subanji, Dr., M.Si., S.Pd. is currently an associate professor in the Department of Mathematics at the Universitas Negeri Malang. He holds a S.Pd. in Mathematics Education from IKIP Malang; a M.Si. in Mathematics from Institut Teknologi Bandung (ITB) and a Dr. in Mathematics Education from Universitas Negeri Surabaya (2007). His research spans the field of mathematics education, especially in thinking process reasearch.



Santi Irawati, Ph.D., M.Si., Dra. is currently an associate professor in the Department of Mathematics at the Universitas Negeri Malang. She holds a Dra. in Mathematics Education from IKIP Malang ; a M.Si. in Mathematics from Institut Teknologi Bandung (ITB) and a Ph.D. in Mathematics from Hyogo University of Teacher Education, Japan (2007). Her research spans the field of mathematics education and mathematics, especially in algebra.