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Self-investigation exercises: geometry learning strategies for prospective teacher students

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ABSTRACT

This study's purpose was to analyse the relationship between geometric thinking skills with self-regulated learning and with students' basic geometry skills. Furthermore, analyze student errors in solving geometry problems based on their self-regulated learning to determine the optimal geometry learning environment for geometry thinking skills. The study subjects were 46 second-semester students majoring in Mathematics Education for the academic year 2021/2022 of Universitas Bina Bangsa who were selected with purposive sampling techniques. This research uses quantitative and qualitative research methods. Data were obtained from geometric thinking ability tests, basic geometry skills tests, self-regulated learning questionnaires, and interview sheets. The results showed a relationship between the ability to think geometrically and the basic geometry skills of students in solving geometry problems in terms of student self-regulated learning. It was found that students' difficulties in solving geometry problems can be overcome by creating an independent geometry learning environment. The learning process design is an independent investigation exercise involving relevance, assurance, motivation, investigation, evaluation, and satisfaction.

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1. INTRODUCTION

Mathematics is a subject that has an important role both in its application in everyday life and the development of other sciences. Geometry is an integral part of mathematics, not only at the national level but also at the international level [1]. Karapınar and Alp İlhan [2] stated that geometry provides a complete appreciation of our world, and geometry plays a vital role in one's ability to understand other concepts. Haver and Vojkůvková [3] also wrote several fundamental reasons for studying geometry, including exploration of geometry can develop one's ability to solve problems, and geometry plays a vital role in understanding other concepts in mathematics. Geometry is a material that has an excellent opportunity to be understood because students have known geometry ideas since before they enter school and are around and in their daily lives. However, based on previous research results, students have low geometric achievement. Geometry achievement is closely related to students' geometric thinking skills [4]. The ability to think geometry is the ability of students to observe objects, build definitions based on the characteristics inherent in objects, recognise the relationship between one object and another, and apply them in solving geometry problems [5]. Based on the results of previous research, it is known that students' ability to think geometrically is still relatively low. Based on the research results from several previous studies, that the ability to understand geometry was in the lowest position of learning achievement [5], [6]. Furthermore, Gridos *et al.* [7] has argued that the importance of geometry as a vehicle for developing different ways of thinking in mathematics.

The cause of students' low geometric thinking skills is the learning process that does not pay attention to the level of students' geometric thinking skills and ignores how to train students' learning independence in solving geometry problems. Regarding the level of geometric thinking ability, Van Hiele [8] wrote that a child with a low level of geometric thinking is unlikely to understand the material at a higher level than the child. Md Yunus *et al.* [9] stated the importance of achieving sequential concepts starting from the initial level of geometric thinking skills because it becomes the basis for later levels. That is, if a child with a low level is forced to understand the material at a level above it, then the child is only at the stage of memorization, not at the stage of understanding. So it is essential to adjust the level of development of children's geometric thinking in designing learning activities. Furthermore, it is related to student learning independence or “self-regulated learning”, Broadbent and Poon [10] mention that someone accustomed to being independent in learning will tend to be calm and meticulous in completing the tasks given. Mathematics is not separated from everyday human life. Therefore mathematics is inseparable from the independence of learning students to solve everyday human problems optimally. Someone with independence can stand alone confidently to face the problems in front of him. Self-regulated learning is not a mental ability or academic skill, such as thinking creatively or reading fluently, but a self-directed attitude is driven by the motivation to achieve goals [11]. It has been a common assumption for a long time that students tend to dislike mathematics lessons because they believe that mathematics is synonymous with complex and scary formulas. Negative assumptions about mathematics impact students who feel low intelligence, so they become unconfident and not motivated to learn independently and tend not to try to learn and solve problems independently. Self-regulated learning does not mean learning alone, but learning that requires a student's independence to be responsible, initiative, and courage in learning without the control of others. According to Wolters and Hussain [12], there are seven indicators of self-regulated learning, namely: i) motivation (learning initiative); ii) cognitive (diagnosing learning needs); iii) setting learning goals; iv) metacognitive (monitoring, organizing and controlling learning); v) self-efficacy (viewing difficulties as challenges); vi) time management strategies and learning environments (choosing and establishing appropriate learning strategies); and vii) evaluation (evaluating learning processes and outcomes).

Furthermore, Broadbent and Poon [10] mentions ten strategies in self-regulated learning, namely: i) self-regulated learning strategies combined; ii) metacognition; iii) time management; iv) effort regulation; v) peer learning; vi) elaboration; vii) rehearsal; viii) organisation; ix) critical thinking; and x) help-seeking. This shows that student self-regulated learning is a process of learning activities, and some strategies can be done in learning activities to stimulate students' self-regulated learning in solving tasks and math problems so that their ability to solve problems improves. Self-regulated learning significantly affects the learning outcomes of students, where learning outcomes determine the quality of learning. However, facts in the field show that the self-regulated learning of prospective teacher students is still relatively low. It is known from the results of previous research that students have low self-regulated learning which is detected from the low ability of students to manage their study time so that it affects academic results [13]. Furthermore, Alghamdi et al.'s research [14] found that students' self-regulation skills were poor. They were unsure of their abilities and were less motivated in the learning process. There is a relationship between the ability to solve geometry problems, basic geometry skills, the level of geometric thinking ability, and student self-regulated learning. Each level of geometric thinking ability has different basic geometry skills. The higher the students' geometric thinking skills, the higher their ability to solve geometry problems. Furthermore, the more independent students are in learning, the higher their ability to solve geometry problems. In line with this, according to Van Hiele [8] stated that the basic geometry skills possessed by students vary in solving geometry problems based on the level of geometric thinking. Hoffer categorizes basic geometry skills into five skills: visual, verbal, drawing, logical, and applied [15]–[19]. Furthermore, some studies [3], [8], [20], [21] stated that the ability to think geometrically is divided into five levels, namely level 0 (visualization), level 1 (analysis), level 2 (abstraction), level 3 (deduction), and level 4 (rigor).

The results of previous research show that the level of geometry thinking of students based on van Hiele's theory is generally the highest only at level 1 (analysis) [22]. This is also supported by [23], that students cannot explain the characteristics of rectangular shapes because the tendency to memorize formulas becomes the primary guideline in solving geometry problems. Furthermore, from several research results [2], [24], [25] was concluded that the level of geometry thinking van Hiele students mostly only reached the level of visualization, and there was a small number who were able to achieve the highest at the level of informal analysis and deduction of research. Furthermore, the results of Decano's research [26] found that most students were identified as concrete operational thinkers, namely at the level of deductive thinking. According to him, students must at least be able to think deductively to study Geometry and mathematics in general successfully. So it becomes essential to analyze in depth the ability of students to solve geometry problems in terms of the level of geometric thinking skills to improve the learning process that is commonly done so far. Knowing what basic geometry skills students have at each level of geometric thinking ability will make it easier to design learning activities that are by the child's level of development and adjusted to his level of thinking. The benefit for students is that the learning process can enrich their experience and thinking so that improving students' geometric thinking skills is more optimal in every meeting.

The results of midterm exams (UTS) students of Mathematics Education at Universitas Bina Bangsa in the second semester of the introductory geometry course, semester IV of the space analytic geometry course and students of the VI semester of the transformation geometry course for the 2021/2022 academic year showed that the average learning outcomes were less than the maximum completeness criteria value (<75), which did not meet expectations and showed problems. The results of UTS students and student interviews are presented on the difficulties faced in solving Geometry course problems shown in Table 1. Table 1 shows that many students have difficulty understanding geometry learning and solving geometry problems. According to students, the difficulties they experience due to geometry are complex and abstract subjects. This assumption needs to be a concern for educators to design a geometry learning environment that can change this assumption. The learning environment can be designed by fostering students' confidence to independently solve geometry problems by presenting abstract concepts from natural objects, namely contextually. In practice, it can be seen from errors in students' work in solving UTS geometry problems given. Students make mistakes because they experience difficulties, including in forming accurate fundamental constructions, accuracy in measurement, long and incomplete time in proof, and obstacles in proving the answers. Noto *et al.* [27] has argued that making mistakes is natural but needs to be followed up immediately not to hurt students. Basic geometry, analytical geometry, and transformation geometry courses are interrelated, so they need to be overcome early so these errors are not sustainable. Analyzing the tendency of mistakes made by students in solving geometry problems can be a clue to determining a suitable learning environment so that these mistakes do not repeat and optimally improve students' geometric thinking skills.

Table 1. Midterm exam results ($\bar{x}$) for students in semesters II, IV, and VI for the 2021 academic year

Semester	Courses	$\bar{x}$	Student difficulties based on interviews
II	Basic geometry	69	- Have not understood the basic concepts of geometric builds and their properties. - Have not understood the relationship between the concepts of geometric builds and their properties (memorize some geometric wake properties but sometimes often inverse between the properties of one geometric construct and another). - Difficulty in solving the problem of proving the nature of geometric constructs. - Often make systematic mistakes in solving geometry problems.
IV	Space analytic geometry	63	- Difficulty in abstractly seeing the location of the distance of the lines in space. - Difficulty determining whether two lines are correct or not formed on the plane of space. - The difficulty of making a simple sketch is in the form of a flat plane based on the problem of the story of the plane of space. - Often make mistakes in constructing images based on story questions.
VI	Geometry transformations	65	- Difficulty in proving the theorem. - Difficulty describing the answer to prove the theorem. - Difficulty understanding the symbols. - Lack of accuracy in describing problems and solving them.

The Newman procedure is one of the tools to analyze the description answer errors made by students in solving geometry problems. Some previous studies [28]–[30] have suggested that the framework of the Newman procedure has five types of errors: reading (or decoding), comprehension, transformation, process skills, and encoding. Mistakes made by students can inform lecturers about progress and what shortcomings still have to be learned. In addition, knowing students' mistakes in solving geometry problems will be considered in designing the learning process and the following learning environment to minimise these mistakes. So that the purpose of this study is to analyze the relationship between the ability to think geometry with the basic geometry skills of students in solving geometry problems and, in terms of student self-regulated learning, analyze student errors in solving geometry problems as a guide to determine the optimal learning environment for geometric thinking skills. The benefit of research is as information for teachers in designing a geometry learning environment that is by students' basic geometry skills so that they can optimally improve their geometric thinking skills.

2. METHOD

This research uses quantitative and qualitative research methods, in the first stage using quantitative methods to obtain quantitative data and then in the second stage using qualitative methods to deepen, expand, and prove quantitative data. Mertler [31] has argued that quantitative research relies on the collection and analysis of numerical data to measure, describe, explain, or predict, as well as make broad generalizations. Carter *et al.* [32] has argued that qualitative research is interactive research in which researchers engage in continuous and continuous experiences with participants, this involvement will later raise a series of strategic, ethical, and personal issues in the qualitative research process. In quantitative research, the research subjects were 46 second-semester students majoring in mathematics education for the 2021/2022 academic year of Universitas Bina Bangsa. The subject was chosen using purposive sampling techniques, where the researcher chose the subject with the consideration that the second-semester students are students who have received geometry learning and will get advanced geometry learning in semesters IV and VI and will become teachers who will teach geometry subjects later. In qualitative research, the research subjects also used purposive sampling techniques selected from 46 previous students with consideration based on the level of geometric thinking skills and the category of student self-regulated learning.

The instruments used in this study are: i) test 1, a test of students' geometric thinking ability level in the form of 25 multiple-choice questions declared valid for use. The test is based on five levels of geometric thinking skills whose indicators are synthesized based on several theories [8], [21], [33]. There are five levels of geometric thinking skills, namely level 1 (visualization), level 2 (analysis), level 3 (informal deduction), level 4 (deduction), and level 5 (rigor), each of which consists of five indicators; ii) test 2, a primary geometry skills test in the form of five description questions made based on indicator synthesis according to several experts that have been declared valid for use [15], [16], [19]; iii) the student self-regulated learning questionnaire consists of 15 questions developed from 7 indicators of self-regulated learning according to Wolters and Hussain [12], and has been validated by experts; and iv) independent interview sheets, made not arranged systematically but in the form of problem outlines, namely to dig deeper into information on student errors in solving geometry problems along with what information students need to make it easier to solve geometry problems (test question 2). The indicators of basic geometry skills used in this study are presented in Table 2.

Table 2. Basic geometry skills grid

Skills	Indicators
Visual	- Observing the properties and features of a flat wake. - Classifying geometric constructs according to the observed features. - Visualize the geometric representations implied by the data by adding supporting elements to solve the problem. - Inferring more information from visual observations.
Verbal	- Visualizing geometric constructs according to their verbal descriptions. - Reveals the properties of flat wakefulness. - Expressing relationships between wake fullness based on wakefulness properties. - Articulate observed patterns and make explanations used in an evidence.
Drawing	- Re-sketching the image and captioning it according to its verbal description. - Finding relationships from flat wake properties to solve problems. - Add functional supporting elements of a wake to solve the problem. - Apply formulas and supporting geometric models that are formed and solve problems.
Logic	- Recognizing the differences and similarities between geometric constructs. - Categorize flat wakes according to their characteristics and properties. - Using logical evidence to determine whether an image enters or does not enter into a particular relationship. - Develop evidence to infer from the information provided.
Applied	- Sketching geometric models based on their physical objects. - Applying the properties of a geometric model in problem-solving. - Develop mathematical models to solve problems. - Apply geometric models in problem-solving.

Data collection was carried out as follows: i) test 1, which tests the level of geometric thinking ability carried out independently by research subjects on May 10, 2022. Test scoring criteria developed by Usikin [34], is that each level has five questions based on indicators. The level grouping is that students are referred to a certain level if they can answer at least 3 out of 5 questions at each particular level correctly; ii) self-regulated learning questionnaire given to research subjects after being given test 1 on May 10, 2022. The measurement scale on the questionnaire uses the Likert scale, where students are asked to choose 1 of 5 answer choices that they think best suits them, namely strongly agree, agree, neutral, disagree, and strongly disagree. The category of student self-regulated learning is grouped into several categories according to Ulandari *et al.* [35], which is attached to Table 3; iii) test 2, which is an actual geometry skills test, is given to selected research subjects based on the results of test 1 and the results of the student self-regulated learning questionnaire, namely one student selected from each level of geometric thinking ability and the level of student self-regulated learning. Test 2 was conducted independently on May 17, 2022. The student's answer sheet is then analyzed based on indicators of basic geometry skills and errors written in solving geometry problems; and iv) interviews were conducted with selected students from each level of geometric thinking ability and level of student self-regulated learning separately on May 24, 2022, to obtain more in-depth information after completing test question 2.

Table 3. Self-regulated learning level

Number	Conversion value		Category
	Score	Value	
1.	76-100	A	Very good
2.	51-75	B	Good
3.	26-50	C	Good enough
4.	0-25	D	Not good

Quantitative data analysis techniques use percentage formulas, linear regression tests and correlation tests, but previously, normality and linearity tests were carried out. Furthermore, quantitative data analysis techniques include data reduction, data presentation, and conclusion drawing or verification [36]. The stages are: i) data reduction, namely correcting test results and student self-regulated learning questionnaires, then grouping students based on their categorization to be research subjects. The results of the work of students who are the subjects of the study will be analyzed for their basic geometry skills and reviewed from their self-regulated learning and then transformed as material for interviews; ii) data presentation, data presentation is carried out in the form of a table, namely the results of students' work in test 2 about basic geometry skills based on the level of geometric thinking ability and the level of independent learning of students, and also explaining their mistakes in doing geometry problems according to Newman's procedures, then the results of interviews with students based on the outline of the research problem; and iii) concluding is concluding data obtained from reducing and presenting data to answer the formulation of research problems.

3. RESULTS AND DISCUSSION

3.1. Analysis of the relationship between self-regulated learning and geometric thinking skills

3.1.1. Test 1 result (geometry thinking ability level test)

The results of test 1 are a test of the level of geometry thinking ability from 46 second-semester students majoring in mathematics education presented in Figure 1. Figure 1 shows that the geometric thinking ability of college students are not sequential starting from level 1 (visualization), level 2 (analysis), level 3 (informal deduction), level 4 (deduction), and level 5 (rigor), but are abilities that college students can have randomly in contrast to the statement of Rofii *et al.* [37] that the level of thinking ability of students is passed sequentially. When students are at a level, they can think geometrically at that level and the previous level. Students are said to be able to have one level if they can answer at least 3 out of 5 questions at that level [8]. From Figure 1, it is known that a fascinating fact is that each student is not "located" at a certain level of geometric thinking ability but is said to be able to have specific geometric thinking skills. Suppose student "a", based on Figure 1, falls into the categories of level 1 (visualization), level 2 (analysis), level 3 (informal deduction), and level 5 (rigor). According to Van Hiele [8] theory, the categorisation of levels is that if students reach level 3 and cannot reach level 4, then student "a" is at level 3. However, in reality, student "a" can meet the level 5 category even though it does not meet the level 4 category. So, in this case, student "a" is said to have the ability to think geometry, visualization, analysis, informal deduction, and rigor.

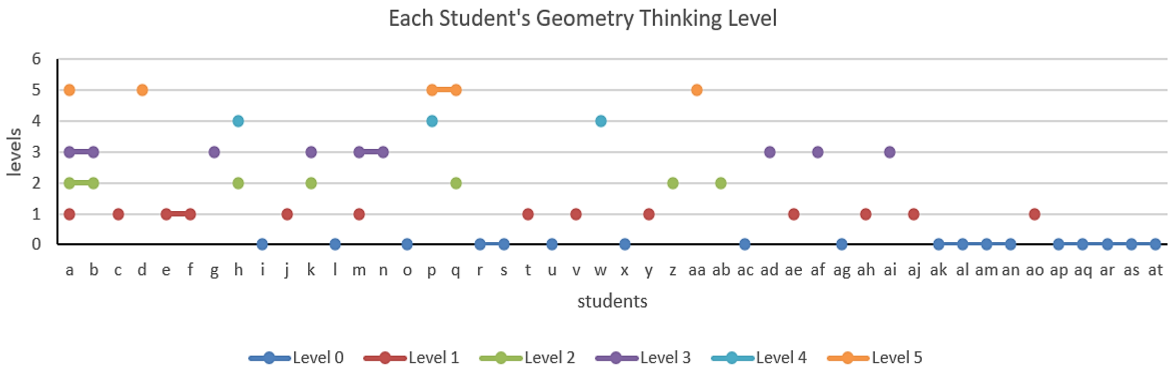


Figure 1. The level of geometry thinking ability of each student

Many other students experience cases like this, according to the facts shown in Figure 1. Another case is students "d" and "aa", who meet level 5 categories but cannot meet levels 1 to 4. Based on this fact, students "d" and "aa" are said to be able to think rigor in geometry. Furthermore, students "h" meet level 2 and 4 categories, meaning that these students can think geometry, analysis, and deduction. Based on these cases, it is concluded that for college students, the ability to think geometry is not level 1 to level 5 that students can achieve sequentially, but is the ability to think geometry that students can have randomly.

Furthermore, to make it easier to read the data, the test result 1 data is presented in Table 4. Table 4 shows that most students (39%) cannot meet the level 1 to 5 category, which is then mentioned as being in the category of level 0. This shows that 39% of students do not have geometric thinking abilities, namely visualization, analysis, informal deduction, deduction, and rigor. Furthermore, it is known that the least number of students is at level 4 (deduction), which is 6.52% of students, meaning that only 6.52% of students can think of deduction geometry. Most students are at level 1 (visualization), which is 28.26% of students, meaning that 28.26% can think in visualization geometry. One of the indicators at level 1 (visualization) is being able to collect geometric construct information based on visuals, and one indicator at level 4 (deduction) is using evidence and theorems to decide the truth value of a mathematical statement. So it can be concluded that, in general, many students are weak in proof but already know the shape of the geometric construct visually. Previous research has similarly found, that many students have a visual understanding of geometric constructs, but they may lack strong mathematical proof, which requires them to explain and outline steps in detail. This can be due to differences in abstract and logical thinking skills [38], [39].

Table 4. Number of students who have a certain level

Geometry thinking levels	Number of students	Percentage of 46 students (%)
0	18	39.13
1 (visualization)	13	28.26
2 (analysis)	7	15.22
3 (informal deduction)	9	19.57
4 (deduction)	3	6.52
5 (rigor)	5	10.87

3.1.2. Results of the self-regulated questionnaire

The student self-regulated learning questionnaire results were given after test 1, namely, the geometric thinking ability level test, to 46 second-semester students majoring in mathematics education, presented in Table 5. Table 5 shows that the percentage of students' self-regulated learning has not met the "excellent" category. Of the seven indicators, there are three indicators with the "good" category, namely cognitive, setting learning goals, and time management strategies and learning environments. Furthermore, there are four indicators in the "good enough" category: motivation, metacognitive, self-efficacy, and evaluation. From this data, it is generalized that student self-regulated learning has not been maximized and can still be improved to a very good level.

Table 5. Percentage of student self-regulated learning based on indicators

No	Self-regulated learning indicators	Score		Category
		Average	Percentage (%)	
1.	Motivation	2.48	49.57	Good enough
2.	Cognitive	3.63	72.61	Good
3.	Setting learning goals	3.74	74.78	Good
4.	Metacognitive	2.5	50	Good enough
5.	Self-efficacy	2.33	46.52	Good enough
6.	Time management strategies and learning environments	3.72	74.35	Good
7.	Evaluation	2.48	49.57	Good enough

Increasing a person's learning independence can greatly increase their confidence in solving the problems at hand. When a person learns to cope with challenges and tasks independently, they develop critical skills such as problem-solving, analysis, and creative thinking. This in turn can strengthen their confidence in dealing with situations that require independent thinking and action. The importance of creating an independent learning environment is that it can help a person deal better with obstacles and failures. They will be more inclined to try new things and take initiative in learning. It can also have a positive impact on their motivation to continue learning and improving. Based on the statement of Anthonysamy *et al.* [40], the better a person's self-regulated learning, the higher his confidence in solving problems and understanding the material being presented. Yan [1] has argued that students who can see their abilities would show an independent attitude and do not need the help of others to complete their tasks. In other words, educators should focus on creating learning activities that can direct students to be independent in learning so that their learning outcomes can be maximized.

Furthermore, the data on the student self-regulated learning questionnaire results are presented in general, as in Table 6. Table 6 shows that most students are at level B with the category "good", and only 8.70% of students with the category "very good" have self-regulated learning. Further analysis was conducted by conducting interviews with several students representing each category. The interview results concluded that students in the "good enough" category do not have a fixed study schedule at home. According to him, learning is an activity that requires a companion from a lecturer. This assumption causes students to depend on others, especially their lecturers, to start learning, which undoubtedly impacts their academic abilities. At the same time, students in the "good" category are known to be enjoyable based on the results of interviews. The "good" category means that students are classified as independent students in learning. However, the results of student interviews admit that they still do not believe in their abilities and tend to see their friends who are more competent in solving geometry problems, and according to him, when they find a difficulty, they student is still afraid to ask questions and not infrequently the problem is not finished. In other words, the "good" category is not necessarily a category of safe self-regulated learning, so it still needs attention to hone student self-regulated learning to be higher.

Table 6. Percentage of student self-regulated learning by level

No	Self-regulated learning level	Category	Number of students	Percentage (%)
1.	A	Very good	4	8.70
2.	B	Good	34	73.91
3.	C	Good enough	8	17.39

Furthermore, students in the "excellent" category mentioned very inspiring things. According to him, learning is not only at school but must continue to be repeated at home to understand the material. The student also mentioned that group study is one way to deepen his understanding because he is required to teach less able friends. This shows that the student is confident in his abilities because he feels he can teach his friends during discussion activities. Furthermore, it is known that the student has a fixed schedule at home and is always on time to study at his study desk. Another fact is that these students are active in class and always try to display their wrong or right abilities. The conclusion is that one way to foster self-regulated learning in students is to foster discipline in learning, invite students to be active with varied classroom activities and foster a sense of responsibility for the tasks given to students.

3.1.3. Regression test results between self-regulated learning and geometric thinking ability

Before conducting the regression test, a normality test will be carried out from both data, namely geometric thinking ability data and student self-regulated learning questionnaires, using data normality tests. The normality test used is the Kolmogorov-Smirnov test, with a significance level of 5% using SPSS Software. The results are presented in the following Table 7. Table 7 shows the normality test results of both data with a significance value of 0.101, which is greater than 0.05. In other words, it is known that H_0 is accepted, which means that the data of self-regulated learning and the ability to think geometry is normally distributed.

Furthermore, a linearity test will be carried out between self-regulated learning data and geometric thinking ability with a significance level of 5% using SPSS Software. The results are presented in the following Table 8. Table 8 shows the results of the linearity test with the value of Sig. for deviation from linearity, which is 0.655. If the value of Sig. is more significant than 0.05, then H_0 is rejected, and it is concluded that there is a significant linear relationship between self-regulated learning and students' geometric thinking ability.

Furthermore, a regression test was carried out to determine the effect of self-regulated learning on the ability to think geometrically with a significance level of 5%. The calculation results are presented in the following Table 9. Table 9 shows the regression test results with a Sig value of 0.000 which is less than 0.05. The conclusion is that H_0 is rejected means that self-regulated learning has a significant influence on students' geometric thinking ability.

Furthermore, the regression equation is determined with the results in Table 10. From Table 10, a constant value of -9.145 is obtained while the regression coefficient value is 0.511. This result is then made by the regression equation: Y=0.511-9.145. The meaning of this regression equation is that it is known that the value of the coefficient is positive, which means that learning independence has a positive effect on students' geometric thinking skills.

Table 7. Data normality test results self-regulated learning and geometric thinking ability

	Kolmogorov-Smirnov[a]		
	Statistic	Df	Sig.
Self-regulated learning	0.112	46	0.101*
Geometric thinking ability	0.085	46	0.101*

a. Lilliefors significance correction
*. This is a lower bound of the true significance

Table 8. Results of the linearity test between self-regulated learning and geometric thinking ability

			Sum of squares	df	Mean square	F	Sig.
Self-regulated learning* geometric thinking ability	Between groups	(Combined)	7,890.35	23	451.025	0.785	0.511
		linearity	2,765.111	1	2,765.111	4.761	0.023
		deviation from linearity	5,331.105	22	283.615	0.561	0.655
	Within groups		7,991.521	23	468.443		
	Total		15,881.871	45			

Table 9. Recapitulation of regression test results between self-regulated learning and geometric thinking ability

Model		Sum of squares	df	Mean square	F	Sig.
1.	Regression	663.211	1	663.211	41.112	0.000[b]
	Residual	431.187	44	17.465		
	Total	1,094.398	45			

a. Dependent variable: Geometric_thinking_ability
b. Predictors: (Constant), self-regulated_learning

Table 10. Regression equation

Model	Unstandardized coefficients B	Unstandardized coefficients Std. Error	Standardized coefficients Beta	T	Sig.
1. (Constant)	-9.145	3.611		-2.311	0.001
Self-regulated_learning	0.511	0.071	0.788	6.601	0.000

a. Dependent variable: Geometric_thinking_ability

3.1.4. Correlation coefficient test results

The correlation coefficient test analyses how closely self-regulated learning relates to geometric thinking ability. The calculation results are presented in the following Table 11. Table 11 shows the results of the Pearson correlation coefficient test between self-regulated learning and geometric thinking ability, which is 0.788. The conclusion is that the relationship between self-regulated learning and the ability to think geometrically is solid. Previously, it was known that the correlation coefficient was positive, which means that self-regulated learning and the ability to think geometry have a positive relationship, meaning that the higher the independence of student learning, the higher the student's geometric thinking ability will also be. In Table 11, it is also known that the determination value of the correlation coefficient is 62.1%, which means that self-regulated learning affects students' geometric thinking skills by 62.1%.

Table 11. Coefficient of correlation between self-regulated learning and geometric thinking skills

Model	R	R square	Adjusted R square	Std. an error in the estimate
1	0.788[a]	0.621	0.598	4.251

a. Predictors: (Constant), self-regulated_learning

3.2. Analyze the relationship of basic geometry skills with geometric thinking ability

The results of test 2 are basic geometry skills tests given to selected subjects, namely one student, each representing variations in the level of geometric thinking ability. From the results of test 1, Table 12 is presented to see the number of students of each level variation and the chosen subject that represents. From Table 12, it is known that 12 selective students represent each of the level variations of geometric thinking ability that are the subject of the study.

The results of test 2, namely the basic geometry skills test of students employed by 12 selected students, are presented in Table 13. Table 13 shows the relationship between students' geometric thinking skills and basic geometry skills. The level of geometric thinking ability is: level 1 (visualization); level 2 (analysis);

level 3 (informal deduction); level 4 (deduction); level 5 (rigor). The provision is that students have one of the primary geometry skills if they meet 3 of the four indicators of each essential geometry skill. In Table 13, there are several interesting facts, namely:

- A level 0 college student only has one indicator each on visual and drawing skills, meaning that students at level 0 do not even have a single essential geometry skill.
- There are similarities in columns 2, 3, and 12. Namely, both have level 1 (visualization) and at least three indicators in the basic skills of visual geometry. From this information, it is known that for students with visualization skills, their visual skills in geometry problems are better than others.
- There are similarities in columns 3, 5, 8, and 12. Namely, both have level 3 abilities (informal deduction) and four indicators of logic skills. In other words, students who can deduct informally then their logic skills are better at solving geometry problems.
- There are similarities in columns 4, 5, 6, 7, and 12: both have level 2 abilities (analytical skills) and at least three indicators of drawing skills. A student with analytical skills, his drawing skills are good in solving geometry problems.
- There are similarities in columns 6, 9, and 10. Namely, both have level 4 (deduction) abilities and at least three indicators of logic and applied skills. A student with deduction skills has better logic and application skills in solving geometry problems.
- There are similarities in columns 7, 10, 11, and 12, which both have level 5 abilities (rigor) and at least three indicators of applied skills. Students with rigor abilities and their applied skills are also good at solving geometry problems.

Table 12. Number of students who achieve variations in geometric thinking ability levels

No	Geometry thinking ability level	Number of students	Percentage (%)	Selected subjects
1.	Level 0	18	39.13	A
2.	Level 1	11	23.91	B
3.	Levels 1 and 3	1	2.17	C
4.	Level 2	2	4.35	D
5.	Levels 2 and 3	1	2.17	E
6.	Levels 2 and 4	2	4.35	F
7.	Levels 2 and 5	1	2.17	G
8.	Level 3	5	10.87	H
9.	Level 4	1	2.17	I
10.	Levels 4 and 5	1	2.17	J
11.	Level 5	2	4.35	K
12.	Levels 1, 2, 3, and 5	1	2.17	L
	Sum	46	100	12

Table 13. Primary geometry skill test results in each level of geometry thinking ability

No	Geometry thinking ability level	Basic geometry skill indicators																			
		Visual				Verbal				Drawing				Logic				Applied			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1.	Level 0	v	x	x	x	x	x	x	x	v	x	x	x	x	x	x	x	x	x	x	x
2.	Level 1	v	v	v	x	x	x	x	x	v	v	x	x	x	x	x	x	v	x	x	x
3.	Level 1 and 3	v	v	v	v	x	x	x	x	v	x	x	x	v	v	v	v	v	v	v	v
4.	Level 2	v	v	v	v	x	v	x	v	v	v	v	v	x	x	x	x	v	x	x	x
5.	Level 2 and 3	v	v	x	x	x	v	x	x	v	v	v	x	v	v	v	v	v	v	x	x
6.	Level 2 and 4	x	x	x	x	x	x	x	x	v	v	v	v	v	v	v	v	v	v	v	x
7.	Level 2 and 5	v	v	v	v	v	x	x	x	v	v	v	v	v	x	x	x	v	v	v	v
8.	Level 3	x	x	x	x	v	x	x	x	v	v	v	x	v	v	v	v	v	v	x	x
9.	Level 4	v	x	x	x	v	v	x	x	x	x	x	x	v	v	v	v	v	v	v	v
10.	Level 4 and 5	x	x	x	x	v	x	x	x	v	v	v	v	v	v	v	v	v	v	v	v
11.	Level 5	v	v	x	x	x	x	x	v	v	v	v	v	v	v	x	x	v	v	v	v
12.	Level 1, 2, 3, 5	v	v	v	x	x	v	v	x	v	x	v	v	v	v	v	v	v	v	v	v

3.3. Analysis of student errors in solving geometry problems in review of self-regulated learning

The following analysis analyses the mistakes made by students in solving geometry problems, namely in test question 2, reviewed based on the category of student self-regulated learning. There are four categories of student self-regulated learning: excellent, good, good enough, and not suitable. Furthermore, four students were selected, each representing each category. However, in this study, there were no students whose self-regulated learning was in the "not good" category, so the researcher only analyzed three categories: excellent, sound, and good enough. The total number of students analyzed for errors in solving geometry problems based on student self-regulated learning was 12 students. The Newman procedure has five categories of errors: reading, compression, transformation, process skill, and encoding [28]–[30]. The total number of possible errors is 300 from 3 categories of self-regulated learning, each with four students, for five categories of errors according to Newman's procedure and five basic geometry skills questions. The calculation is presented in the Table 14.

Table 14. Number of students who make mistakes based on student self-regulated learning

Newman procedure	Categories self-regulated learning		
	Very good	Good	Good enough
Reading	0	4	12
Comprehension	0	5	19
Transformation	3	9	20
Process skill	3	13	20
Encoding	7	19	20
Sum	13 (4.33%)	50 (16.67%)	91 (30.33%)

Based on Table 14, it is known that in doing the basic geometry skills test questions. There were 30.33% of mistakes made by students with "good enough" self-regulated learning, while those with "very good" self-regulated learning only made 4.33%. This shows that students whose self-regulated learning is "excellent" tend to make fewer mistakes than those who are "not good". To provide information on the importance of good self-regulated learning so that students are more thorough in solving problems and minimising mistakes.

Furthermore, data on student errors in solving geometry problems based on Newman's procedure were presented regarding indicators of basic geometry skills. This is done to learn about students' mistakes on fundamental geometry skill indicators. Table 15 shows that the most mistakes students make are in question number 2, with 48 (80%) errors. Judging from the error category, 11 out of 12 students made mistakes in the transformation and encoding categories. Question number 2 is a matter of basic verbal skills, which means that, in general, students still have difficulty in transforming observed patterns and making explanations used in proof and have not been able to use relationships between buildings based on the properties of the building to solve geometry problems so that they miswrite the final result. At the same time, the minor mistake students make is in the fundamental drawing skills problem, which means that students can solve "drawing" problems more than "verbal" problems. From this case, a way is needed to train students to describe proof based on geometric shapes' properties. The term can be because it is accustomed to being used as a basis for choosing an investigation strategy that must appear in learning, which is to familiarize students with investigating problems by providing geometry problems to prove. Nunaki *et al.* [41] stated that investigating math problems can improve students' problem-solving skills. Investigation of mathematical problems involves students in solving unstructured and complex problems. This helps students develop essential problem-solving skills, such as critical thinking, analysis, and geometric modelling. Students also learn to use various relevant mathematical strategies and tools to solve problems.

In general, it is also known that the encoding error category is the most errors made from the entire question number, which is 76.7% errors. Interestingly, the number of students who make mistakes based on the Newman procedure category is sequential, starting from the minor reading error category, followed by the comprehension, transformation, and process skill categories. Finally, most errors are in the encoding category. This shows that when students start making mistakes in the reading category, it is likely that the student will make mistakes in other categories. The reason is that when students cannot read important information from the question and do not even understand what is asked, it is difficult for students to be correct in doing the next step or concluding the final answer correctly.

Table 15. Number of students who made mistakes based on newman's procedures

Newman procedure	Number of students who made mistakes (n=12)						
	Question no 1 (visual)	Question no 2 (verbal)	Question no 3 (drawing)	Question no 4 (logic)	Question no 5 (applied)	Sum	%
Reading	3	8	1	3	1	16	26.7
Comprehension	5	8	4	4	3	24	40
Transformation	7	11	4	5	5	32	53.3
Process skill	9	10	6	5	6	36	60
Encoding	9	11	7	9	10	46	76.7
Sum	33	48	22	26	25		
Persentage (%)	55	80	36.7	43.3	41.7		

Furthermore, it was analyzed how many students' answers in doing the basic geometry skills test question number 1 were reviewed based on student self-regulated learning presented in Figure 2. Figure 2 is the answer of several students to question number 1, which was selected based on the category of student learning independence to be analyzed for errors. Student answers were selected based on the category of student learning independence, namely student 2 representing the "very good" learning independence category, students 1 and 3 representing the "good" learning independence category, and student four representing the "good enough" learning independence category. In more detail, Table 16 will be presented, which describes the analysis of student answers to question number 1 from Figure 2 according to Newman's procedure and the results of interviews with the students concerned.

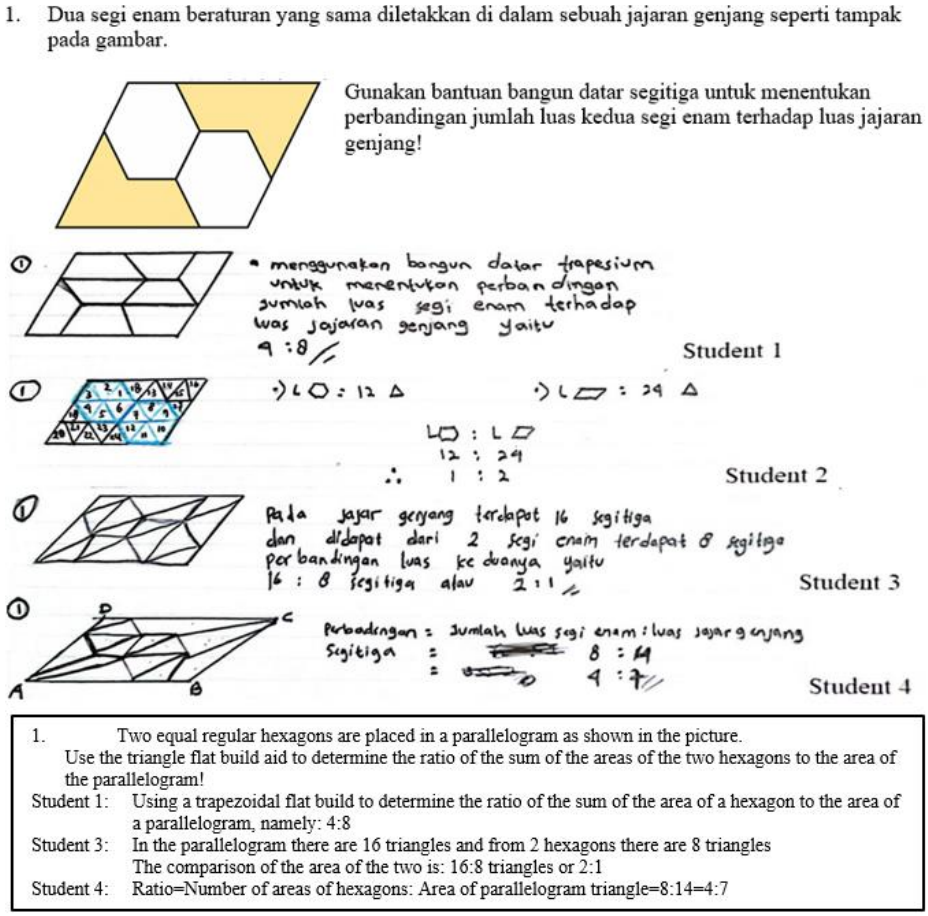


Figure 2. Student answers question number 1

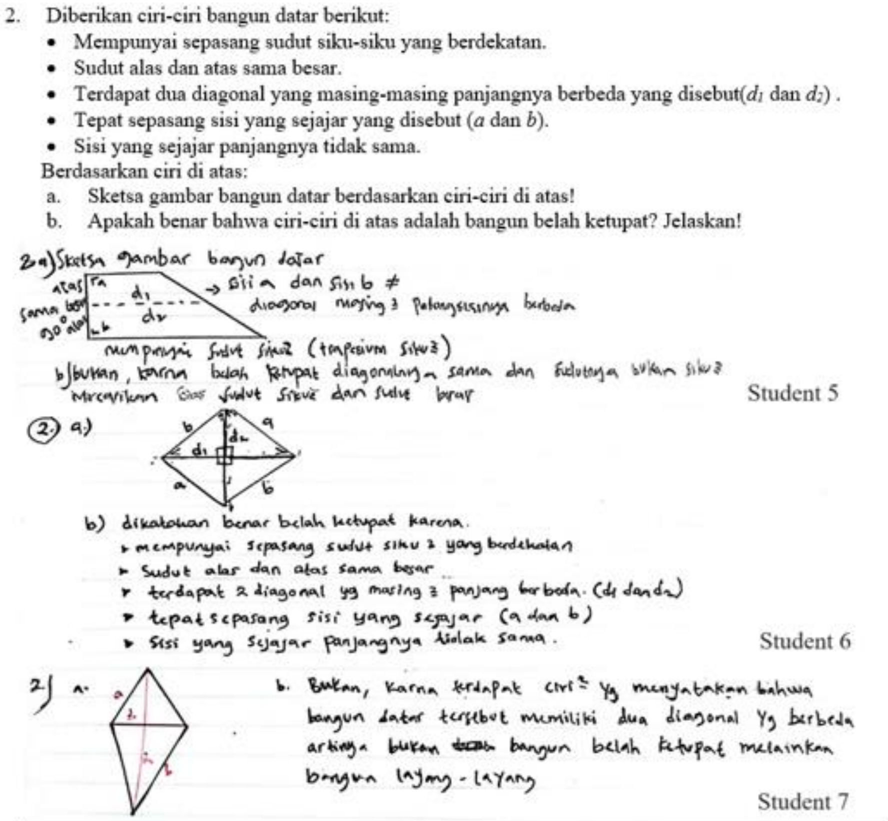
Based on student answer sheets and interview results in Table 16, it is known that on student answer sheet 2, students with "excellent" self-regulated learning give answers carefully and neatly, with the initiative to give numbers to each triangle image formed and go straight to the point of the answer. Furthermore, on the answer sheets of students 1 and 3, namely students with "good" self-regulated learning, give answers that are not wrong but not quite right. This is because the student is not careful. The results of the student interview also mention his mistake in reading the question commands. However, the results of student answers 1 and 3 clearly show that the student understands the concept of the question presented and can give an unordered description. That is, the encouragement of self-regulated learning to students can make students more thorough and minimize their mistakes. A learning strategy that can be used to bring self-regulated learning to students is evaluation, which is to familiarize students to evaluate their work results. Evaluation in solving problems gives students greater responsibility for their learning. According to Siagan *et al.* [42], those accustomed to doing evaluations when solving problems become more independent in identifying and finding solutions to complex mathematical problems. Student answer sheet 4, namely students with "good enough" learning independence, shows students do not understand concepts and lack practice. Students do not give enough effort to elaborate their thoughts in solving problems. So, the answer seems perfunctory and does not match the concept of awakening.

Table 16. Description of student answer analysis on question number 1 based on newman's procedure

Subject	Error categories	Error description and interview results	Analysis
Student 1	Reading	- Misunderstood the meaning of the question in the question, which is to use the help of trapezoidal wakes, while the question of asking to use the help of building triangles. - Misidentifying the information on the question. - Misrepresenting information on the question. - Did not read the question thoroughly, so it miswrote the data known and asked. - Incorrectly apply the procedure according to the problem command, but the steps are correct to resolve the problem.	Student 1 gave the correct final answer but made a mistake reading the question command. In this case, students understand the questions and can solve the questions with the correct procedures and answers. Students' answers also show that students think openly in solving questions. But lack of scrutiny of the questions at hand.
	Process skill	The interview results stated that students claimed to be illegible the phrase "triangular flat wake" on the question and focused on the question to see the comparison of the area of the flat wake.	
Student 2	-	-	Student 2 has correctly answered the question and with the correct procedure according to general directions.
Student 3	Encoding	- Incorrectly concluded the answer. - Not checking the question. The interview results found that the student admitted that he did not carefully read the questions and did not recheck the final answer.	Student 3 has been able to capture information on the questions, master the broad concept of building flats, and can make modelling according to the problem. Student 3 has also been able to operate a flat build area comparison. However, it is wrong to conclude. This is because you do not carefully read what is asked about the question.
Student 4	Process skill	- Incorrectly applying the flat wake area procedure - Incorrectly manipulate a triangular flat wake with different shapes - Wrong to make a conclusion	Student 4 has been able to capture information from the questions and understand what is being asked. Student 4 has also been able to model the area of a flat build. However, student 4 made a mistake at the skill process stage. Student 4 does not yet understand how comparing the area of two flat wakes must be with the help of a flat wake with the same area so that the triangles created are diverse and give incorrect results. As a result, the conclusions made are also wrong.
	Encoding	The results of the interview found that student 4 did question number 1 at the end. The student admitted to being confused by his question. At first, it formed a triangle outside the parallelogram building, and then after a glance at the answer, the friend next to the student tried to draw a triangle inside the parallelogram building. The final result is that the student considers the image formed correctly.	

In conclusion, this student effort can be built with a learning drive inviting students to show their maximum abilities. One of the learning strategies is assurance, which means raising confidence in students by providing problems starting from the easiest. Delivering material and the benefits of learning material (relevance) when learning begins is also one of the strategies that can motivate students to follow the learning process more optimally. Strong self-confidence provides a strong foundation for students to develop themselves. This is supported by Shim and Lee [43] statement that when students believe they can solve math problems, they will be more motivated to learn, practice, and continuously improve their problem-solving skills.

Furthermore, Figure 3 will present some of the students' answers to geometry problem number 2. Figure 3 is the answer of several students to question number 2, which was selected based on the category of student self-regulated learning to be analyzed for errors. Student answers were selected based on the category of student self-regulated learning, namely student 5 representing the "excellent" self-regulated learning category, student 6 representing the "good" self-regulated learning category, and student 7 representing the "good enough" self-regulated learning category. Based on student answer sheets and interview results, it is known that student answer sheet 5, namely students with self-regulated learning "excellent", provide answers with accurate pictures. However, if analyzed, the caption on the picture shows that student five has not fully understood the concept. Student 5 also stated that he was not sure of the answer. This certainly needs to be a concern that students whose self-regulated learning is classified as "very good" also have a sense of insecurity at one particular time. So it is essential to always maintain student confidence during the learning process so that students' learning outcomes are always optimal. One of the learning strategies that can be applied to satisfaction is to give a sense of satisfaction to students by discussing each question practice accompanied by praise for those who present. Satisfaction becomes a positive reinforcement that provides good feedback to students. This helps strengthen the connection between effort and positive results in problem-solving. According to Shim and Lee [43], students will feel encouraged to continue to involve themselves in solving math problems and develop their skills further.



2. Given the following characteristics of flat wake:
- Have a pair of right angles close together.
- The corners of the base and top are equally large.
- There are two diagonals each of different lengths called (d1 and d2).
- Exactly a pair of parallel sides called (a&b).
- Parallel sides are not the same length.

Based on the above characteristics:
a. Sketch a flat drawing based on the characteristics above!
b. Is it true that the above characteristics are rhombic wakes? Explain!

Student 5: a. Sketch of a flat drawing
Side A and Side B =
The diagonal of each side length is different
Has a right angle (right trapezoid)
b. no, because the rhombus is the same diagonally and the angle is not right

Student 6: It is said to be true rhombus because
* has a pair of right angles close together
* The angle of the base and top is equal
* There are 2 diagonals each of different lengths (d1 and d2)
* Exactly a pair of parallel sides (a and b)
* Parallel sides are not equal in length

Student 7: No, because there are characteristics that state that the flat build has two different diagonals, meaning that it is not a rhombus wake but a kite build

Figure 3. Student answers question number 2

Student 6's answer, namely students with "good" self-regulated learning, gave the wrong answer. Student 6's answer erred on "different diagonals" by describing a pair of the same diagonals. This error shows that student 6 is not careful in understanding the problem. The same is done for seven students with self-regulated learning "good enough". Student 7's answer erroneously on the concept of "exactly a pair of parallel sides" gave the wrong picture. The accuracy and understanding of students' geometry concepts in solving geometry problems can be honed using geometry applications. This can also motivate students to follow the learning process (motivation). According to Asigigan and Samur [44], when students are motivated, they tend to be more focused and concentrated on solving math problems. Motivation helps direct their attention to the task and reduces distractions that hinder effective problem-solving.

In more detail, Table 17 will be presented, which describes the analysis of student answers to question number 2 according to Newman's procedure and the results of interviews with the students concerned. In conclusion, students with independence in learning "very well" can solve geometry problems carefully and neatly. Students with the independence of learning "good" and "good enough" have various difficulties including in forming accurate fundamental constructions, accuracy in measuring and understanding the content of the questions, long and incomplete time in proof, and obstacles in proving the answers. Based on the description above, it was found that students' difficulties in solving geometry problems can be overcome by creating an independent geometry learning environment. When a person learns to tackle challenges and tasks independently, they develop critical skills such as problem-solving, analysis, and creative thinking.

It is known that through a creative approach to problems with an independent learning environment helps teachers encourage student creativity in any classroom [45], [46]. However, self-study does not mean that students only learn alone, as Henriksen *et al.* [47] writes, that students need attention to support their creativity during learning. So, it needs the right design so that the learning process with an independent learning environment can have a positive effect on students' thinking skills. The design of the learning process is an independent investigation exercise with steps of relevance, assurance, motivation, investigation, evaluation, and satisfaction, namely: i) delivering material and the benefits of learning the material (relevance); ii) providing problems starting from the easiest, then ask students to prove/solve them independently (assurance/confidence); iii) discuss the problem together, then use the application to prove it (motivation); vi) provide other problems to prove/solve (investigate); v) conduct evaluation; iv) discuss the problem and conclude the material (satisfaction).

Table 17. Description of student answer analysis on question number 2 based on newman procedure

Subject	Error categories	Error description and interview results	Analysis
Student 5	Comprehension	- Misunderstood the term diagonal. - Not knowing the information about the problem thoroughly. The interview results found that students were unsure of the answer. However, the student admitted that he had checked all the characteristics in the image he had made.	Student 5 describes the answer correctly and writes down the correct conclusion. Student 5 understands the questions and can turn the problem information into mathematical form. Nevertheless, student 5 misplaced the signs d_1 and d_2, which are the diagonals of the line. Shows that students do not understand the concept of the diagonal.
Student 6	Transformation Encoding	- Incorrectly changing the question information i the form of an image. - Incorrectly concluded the answer. - Do not check the information on the matter. The interview results are known that students are very confident in their answers. The student admitted that he had checked all the characteristics in his drawings. However, after being asked, "Is the diagonal length of the image the same or different?" the student began to be unsure and stated that he misunderstood the problem.	Student 6 describes incorrect answers and conclusions. This is because it is wrong to understand the meaning of some information on the question. The first is the sentence "a pair of adjacent elbow angles". The student understands that the angle of the elbow must be as intertwined as it depicts. The second is the sentence "two different diagonals" Students understand the concept of diagonals but misunderstand the meaning of "different", so the image created is two diagonals of the same length. The third is the sentence "parallel sides are not the same length" Students understand the meaning of parallel but misunderstand the meaning of "not equal", so the image made is that the parallel sides are the same length. As a result, the result depicted is wrong.
Student 7	Transformation Encoding	- Incorrectly changing the question information in the form of an image. - Misrepresenting the final answer. - Do not check the information on the matter. The interview results found that students were very confident in the answer and crossed out all the features on the drawings. However, after being asked, "Are the a and b sides aligned?" the student began to be unsure but still stated, "Yes, aligned". Then after straightening out, students stated that they misunderstood the problem.	Student 7 describes the wrong answer but describes the correct answer in the conclusion. This answer shows that students are not careful about all the traits given. Students misunderstand the word parallel in one of the properties, namely "exactly a pair of parallel sides", so they assume that line a and line b in the kite image they make are parallel. As a result, the final image created is wrong. However, the reasons expressed are correct, showing that students understand the concept of the material.

4. CONCLUSION

The results showed a relationship between the ability to think geometrically and the basic geometry skills of students in solving geometry problems in terms of student self-regulated learning. The relationship is: i) there is a relationship between the ability to think geometrically with student self-regulated learning with a positive correlation coefficient and robust classification. The higher the student's self-regulated learning, the higher the student's geometric thinking ability in solving geometry problems, especially the seventh indicator of self-regulated learning, namely evaluating processes and results; ii) there is a relationship between the ability to think geometrically with the basic geometry skills of students in solving geometry problems; and iii) based on the error analysis, it was concluded that students with independence learned "very good" more thoroughly in solving geometry problems. Meanwhile, students with independence learn "good" and "good enough" less thoroughly and are not confident in solving geometry problems. Furthermore, based on the description above, it was found that students' difficulties in solving geometry problems can be overcome by creating an independent geometry learning environment. The design of the learning process is an independent investigation exercise with steps of relevance, assurance, motivation, investigation, evaluation, and satisfaction, namely: i) delivering material and benefits of learning material (relevance); ii) providing problems starting from the easiest, then ask students to prove/solve them independently (assurance/confidence); iii) discuss the problem together, then use the application to prove it (motivation); iv) provide other problems to prove/solve (investigate); v) conduct evaluation; and vi) discuss the problem and conclude the material (satisfaction).

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