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A literature review of flexibility in interactive mathematics classrooms: the role of teachers and students

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ABSTRACT

The significance of flexibility in mathematics classrooms, particularly in interactive mathematics classrooms where students learn through hands-on experience, cannot be overstated. Teachers play a pivotal role in fostering students' mathematical understanding by selecting appropriate tasks and providing expert guidance. Additionally, the patterns of interactions between teachers and students, such as elicitation and discussion patterns, and their relationship with teacher flexibility are examined. Developing adaptive expertise in mathematics involves the ability to think flexibly about mathematical concepts, which can be enhanced by teaching math in a way that promotes the development of expertise. Mathematical flexibility, a crucial aspect of mathematical competence, involves the creative and innovative use of mathematical concepts, numerical relations, and strategies. It requires a combination of procedural and conceptual knowledge, skills, and understanding, as well as motivational and affective dispositions. Pedagogical flexibility can significantly contribute to innovating classroom teaching and learning, and exploring the relationships between teachers' pedagogical flexibility and students' potential and actual mathematical flexibility development. The article also emphasizes the importance of developing adaptive competencies in mathematics education to prepare students for an uncertain future, with creativity being considered a key competency and flexibility being viewed as a vital element of creativity.

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1. INTRODUCTION

Flexibility plays a significant role in mathematics classrooms, particularly in interactive settings where students learn through hands-on experiences. Teachers are essential in fostering students' mathematical comprehension by selecting suitable tasks and offering expert guidance. This article also investigates the patterns of interactions between teachers and students in mathematics classrooms, such as elicitation and discussion patterns, and how these patterns relate to teacher flexibility. This is crucial in mathematics classrooms where teachers must be able to adjust their plans and follow unanticipated learning paths initiated by students. An interactive mathematics classroom necessitates a dialogue that encourages learners to actively construct mathematical meaning through communicative processes. Inquiry dialogue is considered a productive environment marked by progressive discussion based on students' need for shared

understanding. The teacher's role is pivotal in promoting students' mathematical comprehension by selecting appropriate tasks and providing them with expert support. The patterns of interactions in teacher-student discussions, such as elicitation and discussion patterns, and how these interactions relate to teacher flexibility, are also examined in this article. Developing adaptive expertise in mathematics involves the ability to think flexibly about concepts. This can be enhanced by teaching math in a manner that promotes the development of expertise through direct and explicit instruction. However, traditional math instruction in the US often relies on cultural scripts that discourage students from creating alternative strategies and may limit their flexibility and creativity. This is known as change resistance and accounts for math development [1].

Mathematical flexibility, an important aspect of mathematical competence, involves the creative and innovative use of mathematical concepts, numerical relationships, and strategies. It requires a blend of procedural and conceptual knowledge, skills, and understanding as well as motivational and affective dispositions. Some of the theoretical, methodological, and educational issues related to mathematical flexibility and present empirical studies in the special issue that address various aspects of mathematical flexibility. There are various perspectives on mathematical flexibility, including strategy use, multiple mathematical representations, and adaptive knowledge. Mathematical flexibility is related to other cognitive and non-cognitive factors such as conceptual understanding, domain-specific factors, and overall mathematical performance [2].

The significance of fostering adaptable skills in mathematics education in order to prepare students for an unpredictable future is highlighted. Creativity is deemed as a crucial competency, while flexibility is regarded as a vital element of creativity. This draws attention to two primary avenues for integrating flexibility into mathematics education: mathematical flexibility (MF) and pedagogical flexibility (PF). MF pertains to altering perspectives on mathematical concepts, processes, representations, and models. PF relates to teachers' decisions to make substantial adjustments to their teaching strategies when designing tasks and planning lessons. Pedagogical flexibility can greatly contribute to innovating classroom teaching and learning, and investigating the associations between teachers' PF and students' potential and actual MF development [3].

There is a relationship between conceptual understanding and procedural knowledge in mathematics, which implies that past studies have mainly concentrated on these aspects independently and have not given sufficient attention to the interplay between them and the role of perceptual processing in problem-solving. The capacity to flexibly perceive mathematical elements is an essential factor in students' access to learned concepts and their choice of mathematical solutions. Perceptual flexibility is the capacity to adaptably perceive mathematical elements, drawing on gestaltist viewpoints, perceptual learning, and visual orientation [4].

The integration of information and communication technologies in education aims to improve university students' capacity to solve engineering problems effectively. To address these concerns, a research program was proposed to enhance mathematics teaching, which included the design of new curricula, the use of computers to support teaching, and the improvement of students' attitudes toward mathematics. A teaching model that focuses on the practical application of mathematical knowledge gained in both study and practice utilizes information and communication technologies (ICT) to equip students with effective tools for problem-solving. The model was tested through pedagogical experiments and included a package of educational e-materials such as interactive teaching materials, e-lectures, and self-tests. The success of the students in the subjects was evaluated through posttest and semester tests, with an emphasis on student activity, creativity, and self-activity. An example of interactive learning material oriented towards the effective use of wxMaxima software in mathematical analyses and algebra is also provided [5].

Students often rely on a single method when solving mathematical problems, and this can limit their flexibility in mathematical reasoning during their school years. The use of manipulative materials can encourage teachers to consider different strategies and promote curiosity and reflection. This study aims to explore how teachers' mathematical flexibility can be fostered and what types of statements promote flexible thinking. Additionally, the study will compare mathematical flexibility in the face of different types of area calculation problems. To achieve these objectives, a review of research on the learning of area magnitude and mathematical flexibility will be conducted [6].

The interactive mathematics classroom, where teachers facilitate learners' active construction of mathematical meaning through communicative processes, has garnered significant interest. This approach equips students with adaptive competencies in mathematics education, preparing them for an uncertain future. A literature review is essential to delve deeper into the theoretical, methodological, and educational issues related to mathematical flexibility and how to foster it in the classroom. This research focuses on developing adaptive expertise in mathematics and promoting mathematical flexibility in alignment with my goals as a mathematics educator. The study aims to investigate the connection between teacher flexibility and patterns of interactions in a mathematics classroom and the necessary knowledge and skills for teachers to become effective and positively impact their students' mathematical development.

2. RESEARCH METHOD

This literature review aims to examine interactive classroom environments in mathematics by incorporating the concept of flexibility. More specifically, the focus is on the pedagogical flexibility of the teacher and mathematical flexibility of the student. The analysis in the discussion section will investigate how this flexible class can be effectively implemented until the teacher is able to assess the learning outcomes, including the use of technology to create an interactive and flexible learning environment.

The search for relevant articles was conducted from August 2023 to October 2023, with electronic databases including Scopus, Google Scholar, PubMed, and Garuda used to gain insight into the trends of transformational leadership and teacher well-being in Indonesia. Search strings consisting of a combination of the terms "mathematical flexibility" and "mathematics" were used, and 95 articles were produced. Of these, 93 articles were screened by reading their titles and abstracts, and 82 articles were excluded, leaving 12 articles that met the criteria. Ten of these 12 articles were assessed for eligibility, with six excluded for not including mathematical flexibility in their interactive mathematics classroom studies.

3. RESULTS

3.1. The role of the teacher in promoting mathematical understanding

3.1.1. Choosing appropriate tasks

The importance of a teacher's role in promoting mathematical understanding and fostering deep knowledge cannot be overstated. Teachers are responsible for selecting and creating tasks that promote student understanding and construction of new knowledge [7]. By choosing challenging yet accessible tasks, teachers can facilitate student engagement and learning. The teacher's knowledge of mathematical tasks is crucial for effectively guiding students and fostering deep understanding [1], [8]. Additionally, teachers can promote mathematical flexibility by making informed decisions about the tasks they assign and by encouraging students to avoid a fixed perspective [3].

3.1.2. Providing expert assistance

The significance of a teacher's role in fostering mathematical comprehension through professional guidance is essential. Teachers have the responsibility to possess a deep understanding of mathematical concepts and effectively guide students in their learning [9]. By having a strong knowledge of mathematical tasks and concepts, teachers can select and create appropriate tasks that promote student understanding and the construction of new knowledge [9]. Additionally, teachers can support students in using clear and concise mathematical language to effectively communicate their understanding [10]. By offering expert assistance and guidance, teachers can facilitate student engagement, foster deep understanding, and promote mathematical flexibility [9].

3.1.3. Developing adaptive expertise

Teachers play a significant role in creating a supportive learning environment that encourages students to explore multiple strategies and approaches to solving mathematical problems. They can facilitate discussions that highlight the importance of flexibility in problem-solving and help students develop a deep understanding of mathematical concepts. Teachers can also provide opportunities for students to engage in tasks that require flexible thinking, such as open-ended problems or real-world applications of mathematics [11]. By providing feedback and guidance, teachers can support students in developing adaptive expertise that involves the integration of conceptual and procedural knowledge [12]. The teacher's role is to guide and scaffold students' learning, helping them become flexible thinkers and problem-solvers in mathematics [11].

3.1.4. The relationship between teacher flexibility and student understanding

The significance of a teacher's role in nurturing mathematical understanding and instilling adaptability to various mathematical situations is paramount [13]. Educators are instrumental in creating an encouraging learning environment that motivates students to explore diverse strategies and approaches for solving mathematical problems [13]. Teachers can make a substantial impact on students' mathematical growth. Furthermore, providing students with tasks that call for flexible thinking, such as open-ended problems or real-world applications of mathematics [1], can help them develop adaptive expertise that merges procedural and conceptual knowledge [13]. By offering feedback and support, teachers can foster students' ability to think flexibly and solve problems effectively [13]. The teacher's role is to facilitate and support students' learning, enabling them to become adaptable thinkers and problem-solvers in mathematics [13]. Research has shown that when teachers utilize evidence-based practices such as promoting student discussion, presenting multiple solution strategies, and employing error analysis, they can enhance students' mathematical understanding [8]. These practices enhance flexibility in problem-solving and deepen students' knowledge [8]. Additionally, creating a supportive and safe classroom environment where mistakes are

valued promotes student understanding [8]. Exposure of mathematics students to thinking creatively and flexibly about mathematical concepts and ideas is also essential [7].

3.2. Patterns of interactions between teachers and students

3.2.1. Elicitation and discussion patterns

The patterns of interaction between teachers and students in elicitation and discussion play a crucial role in promoting mathematical flexibility. To understand these patterns, researchers have analyzed how teachers regulate their elicitation prompts following students' responses to initial mathematical queries [14]. The nature of the interactions between students and between the teacher and students also contributes to the development of mathematical flexibility [15]. The analysis focuses on characterizing situations in which teachers display either flexible or inflexible interactions with students, highlighting the importance of teacher-led discussions in fostering mathematical understanding [16]. These patterns help create a supportive learning environment that encourages students to explore multiple strategies and approaches to problem-solving, promoting adaptive expertise and deep conceptual understanding.

3.2.2. The relationship between teacher flexibility and patterns of interactions

The relationship between teacher flexibility and student learning is significant. When teachers exhibit flexibility in their interactions with students, they create a supportive learning environment that encourages students to explore multiple strategies and approaches to problem-solving. Teachers who respond flexibly to student replies and adapt their plans accordingly can foster mathematical discussions that promote deeper conceptual understanding and adaptive expertise. Conversely, inflexible interactions can hinder students' mathematical flexibility and limit their opportunities for exploration and growth. Therefore, teacher flexibility in orchestrating interactions plays a crucial role in promoting mathematical understanding and developing adaptive expertise [16], [17].

3.2.3. The impact of teacher flexibility on student learning

In conclusion, teacher flexibility positively impacts student learning by promoting mathematical understanding, deepening knowledge, and encouraging flexible thinking and problem-solving approaches. Teachers who employ evidence-based practices, such as encouraging student discussions, presenting multiple solution strategies, and using error analysis, can improve students' mathematical understanding and enhance their flexibility in problem-solving. Additionally, teacher flexibility in orchestrating interactions with students plays a crucial role in fostering mathematical discussions that promote deeper conceptual understanding and adaptive expertise [3]. Overall, teacher flexibility positively influenced student learning by promoting mathematical understanding, deepening knowledge, and encouraging flexible thinking and problem-solving approaches.

Teachers who exhibit flexibility have a positive influence on students' mathematical understanding by promoting comprehension, deepening knowledge, and fostering adaptable thinking and problem-solving strategies. Research-based methods, such as facilitating student discussions and presenting multiple problem-solving approaches, can enhance students' mathematical comprehension. The interactions between teachers and students also play a critical role in promoting mathematical flexibility, as teacher-led discussions create a nurturing learning environment that encourages exploration and multiple approaches to problem-solving. Teacher flexibility in directing interactions can guide and support students' learning, helping them become adaptable thinkers and problem-solvers [3].

3.3. The importance of developing adaptive expertise in mathematics

3.3.1. The ability to think flexibly about mathematical concepts

The ability to think adaptably about mathematical concepts involves employing various strategies or approaches when addressing mathematical problems. Mathematical flexibility enables students to investigate different methods, make connections between diverse mathematical ideas, and modify their thinking based on the problem's context. This includes being receptive to different perspectives, considering multiple possibilities, and being open to revising and refining one's thinking. Adaptable thinking promotes a deeper understanding of mathematical concepts and enhances problem-solving abilities [18].

3.3.2. The relevance of adaptive expertise in real-world applications

The development of adaptive expertise in mathematics is crucial due to its relevance to real-world applications. Adaptive expertise allows individuals to think flexibly about mathematical concepts and apply their knowledge to novel situations and problem-solving challenges [19]. This flexibility fosters deeper conceptual understanding and enables individuals to integrate both procedural and conceptual knowledge [20]. In real-world circumstances, adaptive expertise enables individuals to tackle problems from various

angles, explore different strategies, and adapt their thinking to devise innovative solutions [21]. This skill also enhances problem-solving abilities and promotes mathematical understanding, which are valuable in diverse fields such as science, engineering, finance, and technology.

3.3.3. The benefits of developing adaptive expertise in mathematics education

The benefits of developing adaptive expertise in mathematics education for mathematical flexibility are significant. Adaptive expertise involves flexibility in problem-solving, utilizing multiple strategies, and adaptivity in selecting the most suitable approach. By integrating conceptual and procedural knowledge, individuals can enhance their mathematical flexibility, allowing them to tackle a variety of problems effectively. Cognitive psychology has contributed to a deeper understanding of flexibility and its role in mathematical expertise, emphasizing the value of adaptive expertise for developing versatile mathematical capabilities [12], [22], [23].

Developing adaptive expertise in mathematics education brings several benefits. It allows students to effectively apply their mathematical knowledge and skills in various contexts, enhancing their problem-solving abilities [19]. Adaptive expertise helps students develop a deep understanding of mathematical concepts and strategies, enabling them to transfer their knowledge to real-world situations and new problem-solving scenarios [20]. It also aids in developing flexible thinking and metacognitive skills, allowing students to monitor and regulate their own learning processes [12]. Furthermore, adaptive expertise fosters a positive attitude towards mathematics, promoting confidence and motivation in students [19]. Overall, developing adaptive expertise in mathematics education enhances students' mathematical proficiency and prepares them for success in higher-level math and real-life applications.

3.4. Pedagogical flexibility in mathematics education

3.4.1. The need to prepare students for an uncertain future

Pedagogical flexibility is essential in mathematics education for preparing students for an unpredictable future. As society undergoes rapid changes, educational objectives must concentrate on fostering adaptive competencies instead of merely mastering mathematical algorithms. To achieve this, teachers need to implement significant adjustments to their teaching methods to promote mathematical understanding among students [3]. Research-based practices, such as encouraging student discussion and presenting various solution strategies, are effective in enhancing students' mathematical understanding [3]. Furthermore, developing adaptive expertise in mathematics is critical to think flexibly about mathematical concepts and apply this knowledge to real-world applications [24]. This approach enables individuals to solve problems from different angles, explore various strategies, and adapt their thinking to find creative solutions, ultimately enhancing their problem-solving abilities and promoting mathematical understanding [3]. Ultimately, pedagogical flexibility is a vital component in preparing students for an uncertain future by fostering adaptability and promoting mathematical understanding.

3.4.2. Developing the ability to adapt to different teaching situations

Pedagogical flexibility in mathematics education is essential for developing the ability to adjust to various teaching situations. This allows teachers to customize their teaching methods based on students' unique needs and learning preferences [8]. By employing research-supported techniques such as facilitating student discussions and presenting multiple problem-solving tactics, teachers can enhance mathematical comprehension and deepen students' understanding [3]. Teacher-led discussions establish a nurturing learning environment that encourages exploration and multiple approaches to problem-solving, fostering flexible thinking and problem-solving techniques [3]. This adaptable expertise in mathematics enables individuals to think creatively about mathematical concepts, approach problems from different perspectives, and adjust their thinking to find innovative solutions [3]. In conclusion, pedagogical flexibility in mathematics education prepares students for an unpredictable future by fostering adaptive abilities and enhancing mathematical comprehension [3], [8].

3.4.3. The importance of flexible teaching strategies

Pedagogical flexibility in mathematics education plays a crucial role in the importance of flexible teaching strategies. It enables teachers to adapt their instructional approaches to meet the diverse needs of students. By using flexible teaching strategies, teachers can cater to different learning styles, abilities, and interests, ensuring that all students have the opportunity to engage actively in the learning process. Flexible teaching strategies also promote mathematical understanding by providing multiple entry points and approaches to problem-solving. Teachers can use various instructional methods, such as hands-on activities, group work, technology integration, and real-world applications, to help students develop a deep understanding of mathematical concepts.

Moreover, flexible teaching strategies encourage students to think critically and creatively by presenting them with open-ended tasks and challenging problems. By allowing students to explore different solution pathways, teachers foster flexible thinking and problem-solving approaches. This approach empowers students to develop their own strategies, make connections between different mathematical concepts, and apply their knowledge in new and unfamiliar situations. In summary, pedagogical flexibility in mathematics education is essential for promoting mathematical understanding, fostering flexible thinking and problem-solving approaches, and providing an inclusive learning environment. By adopting flexible teaching strategies, teachers can effectively address the diverse needs of their students and create a dynamic and engaging learning experience.

The pedagogical flexibility in mathematics education is crucial in promoting the importance of flexible teaching strategies. It involves adapting teaching approaches to meet the diverse learning needs of students and prepare them for an uncertain future. By employing evidence-based practices such as encouraging student discussion, presenting multiple solution strategies, and promoting conceptual understanding, pedagogical flexibility enhances students' problem-solving abilities and mathematical understanding [11]. It also fosters the development of adaptive expertise, allowing students to think flexibly, integrate conceptual and procedural knowledge, and approach problems from multiple angles [3]. This adaptability is essential for applying mathematical concepts to real-world applications and preparing students to adapt to different teaching situations [25]. Overall, pedagogical flexibility plays a crucial role in developing adaptive competencies and promoting mathematical understanding.

4. DISCUSSION

4.1. A new teaching model that focuses on the applicability of gained mathematical knowledge in both study and practice

The new teaching model that focuses on the applicability of gained mathematical knowledge in both study and practice for mathematical flexibility is designed to enhance students' adaptive competencies in mathematics education. This model emphasizes the importance of teachers being flexible and adaptable to unexpected learning trajectories initiated by students. It aims to improve students' ability to solve engineering problems effectively by promoting conceptual understanding and procedural knowledge in mathematics. The model evaluates students' success through various tests and encourages the use of information and communication technologies in education to support problem-solving and enhance perceptual flexibility [13].

4.2. The role of teacher in developing this type of teaching model

The role of the teacher in the teaching model to develop mathematical flexibility is crucial. Teachers' flexibility plays a substantial role in facilitating students' mathematical flexibility, emphasizing the importance of creativity and flexibility in teaching for adapting to changing contexts [3]. The study aims to explore the impact of pedagogical flexibility (PF) on students' learning and the relationships between teachers' PF and students' mathematical flexibility (MF) development [3]. It also delves into the conceptualization of teacher curriculum use, distinguishing between official and operative curriculum, and focuses on how PF is used in lesson planning and implementation when teaching Grade 9 students and its impact on students' learning experiences [3]. Flexibility in problem solving is considered an important skill for students, allowing them to generate, use, and evaluate multiple solution methods [13]. This flexibility is closely related to adaptive expertise and has been linked to improvements in students' conceptual and procedural knowledge [13].

4.3. The role of teacher in using information and communication technologies to enhance student learning

The role of the teacher in using ICT to enhance student learning mathematical flexibility is crucial. Teachers' flexibility plays a substantial role in facilitating students' mathematical flexibility and adapting to changing contexts [26]. Teaching mathematics with digital technology is an integral part of classroom practices in many primary, secondary, and tertiary educational settings [27]. Technology provides dynamic opportunities for instruction in math and STEM classrooms, enhancing the learning process and making concepts more engaging and interactive [28].

4.4. The importance of evaluating student success in enhancing adaptive competencies in mathematics education

The importance of the applicability of mathematical knowledge in both study and practice is significant in mathematical flexibility. This concept emphasizes the need for students to be able to solve problems in multiple ways and highlights the importance of procedural knowledge alongside conceptual

understanding [13]. It also emphasizes the relationship between flexibility in problem-solving and adaptive expertise, which influences both conceptual and procedural knowledge [13]. This highlights the significance of students being able to apply mathematical knowledge in various contexts to enhance their problem-solving abilities and adaptability in mathematics education.

Student success can be evaluated in mathematics classrooms through authentic assessment, which focuses on applying mathematical knowledge in real-world contexts, emphasizing analytical skills, creativity, and collaboration [29]. This approach goes beyond rote memorization and multiple-choice tests, assessing students' understanding and application of mathematical concepts, using methods such as performance assessments, short investigations, open-response questions, portfolios, and self-assessment [29]. Additionally, reflective journaling and real-world projects can be used to engage students and provide a more comprehensive view of their math understanding, supporting instructional decisions [30]. Assessment can also be integrated with classroom discourse and activity through observation, questioning, and written discourse [29].

5. CONCLUSION

There is the relationship between teacher flexibility and patterns of interactions in a mathematics classroom. Flexibility is important in interactive mathematics classrooms, where teachers need to be able to follow unexpected learning trajectories initiated by students. The study aims to enhance the adaptive competencies in mathematics education to prepare students for an uncertain future. The relationship between conceptual understanding and procedural knowledge in mathematics is discussed, and the importance of perceptual flexibility in problem solving is highlighted. The implementation of information and communication technologies in education is proposed to improve students' ability to solve engineering problems effectively. A new teaching model that focuses on the applicability of gained mathematical knowledge in both study and practice is introduced, and the success of students is evaluated through various tests.

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