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Development and validation of self-reflected mathematical misconceptions scale

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ABSTRACT

Developing an assessment tool to identify mathematical misconceptions is important for early intervention and support for at-risk students. This exploratory sequential mixed methods study aimed to develop and validate a questionnaire for self-reflection on mathematical misconceptions among senior high school students using exploratory and confirmatory factor analyses as an application of structural equation modeling (SEM). This study involved 80 senior high school mathematics students across regions in the Philippines for the mathematical misconception test in the first phase. Of these, 20 purposively selected students who committed the most errors in the misconception test were interviewed to explore the underlying constructs of the students' mathematical misconceptions. For the third and final phase, 310 selected students completed the developed self-reflected mathematical misconception scale. In this study, we identified four key factors of mathematical misconceptions: lack of procedural and conceptual knowledge, poor mathematical abstraction, internal barriers, and cognitive conflict. The developed scale, comprising 41 validated items, was tested valid and reliable tool for educators in assessing and addressing students' mathematical misconceptions, allowing for designed instructional strategies and targeted interventions. Further research is recommended to explore the causes and remediation of mathematical misconceptions and track students' progress in addressing them over time.

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1. INTRODUCTION

The field of secondary mathematics education is at a critical point where students often need help with mathematical misconceptions that can significantly hinder their understanding of mathematical concepts. These misconceptions have been a common problem, particularly among senior high school students. Misunderstandings and errors in mathematics have gained attention from researchers, scholars, and mathematics educators, leading to numerous studies aiming to comprehend the scope of these issues and develop strategies to lessen their impact.

Ojose [1] highlights the prevalence of misunderstandings and errors in mathematics across student populations. These issues are multifaceted, influenced by various factors such as student attitudes toward mathematics, teaching methods, teaching abilities, students' preconceptions, limited understanding, inadequate modeling, and insufficient higher-order thinking skills [2]–[7]. Mathematical misconceptions often arise from students' prior knowledge, which they erroneously generalize [8]. These misconceptions can lead students to believe their incorrect methods are correct or to feel uncertain about their approaches, eventually resulting in persistent errors [9], [10]. In addition, errors may be committed by students due to carelessness when verifying answers [11].

Various studies have focused on identifying specific misconceptions within mathematical concepts and have emphasized the importance of addressing them proactively. Problem-solving activities have emerged as a practical approach to pinpoint and rectify these misconceptions [12]. Furthermore, understanding the underlying reasons for these misconceptions is essential. Visual mathematics has been proposed as a pedagogical approach to help students grasp mathematical concepts better, as students often develop misconceptions when taught to memorize procedures rather than understand underlying principles [13]. In addition to addressing misconceptions, the literature highlights the importance of self-reflective thinking in mathematics education. Self-reflection can foster metacognitive processes, enabling students to become more aware of their thinking processes, identify strengths and weaknesses, and develop strategies for improvement [14]. Significantly, self-reflection can improve students' attitudes toward mathematics, helping them develop a more positive outlook and better performance [15].

This research undertook a distinctive approach to mathematical misconceptions among senior high school science, technology, engineering, and mathematics (STEM) students in the Philippines. Unlike prior studies that primarily aimed to identify misconceptions and administer interventions afterward, this study sought to empower students through self-assessment and self-reflection. Developing a valid self-assessment tool will encourage students to engage with their mathematical understanding, fostering a sense of ownership in their learning process. The utilization of confirmatory factor analysis (CFA) added a layer of methodological sophistication to the study, elevating its reliability and validity.

The primary goal of this research was to equip senior high school STEM students with the tools to recognize and rectify their mathematical misconceptions, potentially enhancing their overall mathematical competence. Additionally, this study holds promise for future researchers and educators, offering a valuable resource for those interested in innovative approaches to address misconceptions and promote self-directed learning in mathematics education. This study aims to develop and validate a self-reflected mathematical misconceptions scale (SMMS) for senior high school students. Specifically, this study sought to answer the following research questions: i) what are the underlying constructs of the SMMS? and ii) does the SMMS possess adequate internal consistency?

2. LITERATURE REVIEW

2.1. Mathematical misconceptions

Misconceptions in mathematics, particularly among senior high school students, pose significant challenges to their comprehension and application of mathematical concepts [1]. Numerous studies have explored the causes of these misconceptions, attributing them to factors such as student attitudes, teaching methods, and limited understanding [3], [5]. Students' inaccurate ideas often stem from a lack of clarity in concept learning, leading to persistent errors and hindering academic performance [8]. Recognizing the distinction between errors and misconceptions is crucial, as errors result from negligence, while misconceptions arise from misunderstandings [16].

Research over the past decade has categorized general mathematics learning mistakes into systematic, random, and thoughtless errors [17]. Misconceptions, as described by Tippett [18], represent disagreements between students and experts, resulting in systematic errors in understanding [19]. In a broader context, inadequate mastery of fundamental mathematical concepts increases the likelihood of employing incorrect strategies, emphasizing the importance of a solid mathematical foundation for effective learning [20]. Thus, educators play a significant role in addressing these misconceptions, requiring awareness of common student misunderstandings and the development of effective strategies.

Several studies propose problem-solving activities as a promising approach to identify and correct misconceptions, emphasizing the importance of real-world applications in mathematics education [21]. Visualizing mathematical concepts, as advocated by Boaler *et al*. [13] is considered essential in overcoming misconceptions caused by rote memorization. The literature emphasizes the challenge of correcting deeply rooted misconceptions and emphasizes the role of technology in providing visual and interactive representations to deepen students' understanding [22]. Addressing misconceptions early in the learning process is important for promoting students' mathematical understanding and performance.

2.2. Self-reflection in mathematics

Self-reflective thinking in mathematics education has garnered considerable attention, with a focus on enhancing students' learning experiences and improving their performance [23]. Self-reflective thinking promotes metacognitive processes, necessary for effective problem-solving in mathematics [24]. Through self-reflection, students gain awareness of their thinking processes, identify strengths and weaknesses, and develop strategies for improvement. An essential aspect of self-reflective thinking is its potential to positively impact students' attitudes towards mathematics [25]. Many students face challenges and harbor negative attitudes towards the subject, but self-reflection can contribute to a more positive outlook. Integrating self-reflective thinking into mathematics instruction, such as through problem-solving courses or reflective journaling, has been shown to promote metacognition and improve overall performance [26]. Additionally, self-reflection facilitates the identification and resolution of misconceptions contributing to improved teaching practices [27].

3. METHOD

3.1. Research design

Exploratory sequential mixed method design was the design used by the researcher. It was a method for sequentially collecting and analyzing qualitative and quantitative data [28]. The research followed a three-phase approach. Initially, qualitative data was collected and analyzed, followed by the collection and analysis of quantitative data. Finally, the data from these two distinct strands were integrated to examine the phenomenon in greater depth and to explain the connection between the qualitative and quantitative findings. The first phase involved qualitative data collection to examine the phenomenon, followed by the collection of quantitative data to elucidate the relationship between the qualitative data.

3.2. Participants

The participants in this research consisted of senior high school students enrolled during the 2022-2023 academic year from various regions in the Philippines, including the national capital region (NCR). Senior high school students were selected for this study because they were still at a stage. The first phase of data collection involved 80 senior high school STEM students from diverse regions. These participants provided the initial dataset for the study. In the second phase, 20 respondents were carefully selected from the first-phase participants based on their performance in the mathematical misconception tests. Specifically, those who made the highest number of mistakes were chosen. The third and final phase expanded the participant pool to 310 senior high school STEM students from different regions. This phase primarily involved completing the SMMS.

3.3. Instruments

The research used a set of instruments for each phase of data collection. The mathematical misconceptions tests were administered to the participants in the first phase. This test covered general mathematics, probability, pre-calculus, and basic calculus competencies. The delivery of the test was facilitated through Google Forms, ensuring ease of access for participants. The second phase used interview questionnaires based on responses obtained in the first phase. These questionnaires were designed to be highly customized, with questions designed to each participant's specific responses from the initial test phase. For the third and final phase, the research introduced the "questionnaire on self-reflected mathematical misconceptions". This comprehensive instrument contained 70 statements aimed at assessing misconceptions among participants. It utilized a Likert scale, allowing participants to rate their responses. The development of this questionnaire was based on insights gained from the first and second phases of data collection.

3.4. Data analysis

The Delphi method was utilized to analyze the data of this study. The Delphi method is a systematic process used to develop a group of opinions collected from a specific group of specialization [29]. This method involved a series of data collection. This study had two phases (first and second rounds) of gathering qualitative data to develop collective opinions that were beneficial for formulating possible statements incorporated into the developing tool. Then, another phase of data collection (third round) for quantitative data, which was pilot testing, was conducted. Exploratory factor analysis (EFA) and CFA were utilized for the quantitative data that was collected from the third phase. EFA was a statistical inference method for developing and validating theories and measurements [30]. The researchers used this statistical method to test the consistency of the items with the generated construct from the data to ensure the reliability of the instrument. The CFA is a statistical technique for psychopathology and personality questionnaire construct validation [31]. The researcher utilized CFA to test the validity of the instrument.

4. RESULTS

4.1. The underlying constructs of the questionnaire on self–reflection on mathematical misconception

To determine the underlying constructs and statements of the questionnaire on self-reflection of the mathematical misconceptions, the researchers used the EFA. Table 1 presents key findings from the analysis.

The kaiser-meyer-olkin (KMO) value of 0.933, exceeding the threshold of 0.50, indicates the instrument's high utility. Furthermore, the p-value of 0.000 in Bartlett's test of sphericity demonstrates the instrument's significance. The total variance explained in the initial iteration reveals insights into the potential number of components that can be derived from the evaluated items. Upon examination, it was observed that the extraction sums of squared loadings and rotation sums of squared loadings columns were empty for items 5 to 70. Four components were identified, focusing on items with complete data in both rows and columns, suggesting that the data evaluation resulted in four factors.

The researchers derived a final KMO value of 0.931 through this iterative process. This value surpasses the minimum threshold of 0.50 and approaches unity, indicating that the instrument employed in the study is valid. Additionally, the significance of the tool was evaluated through Bartlett's test of sphericity, yielding a p-value of 0.000. The obtained p-value signifies the statistical significance of the instrument. Consequently, based on the results mentioned above, the instrument utilized in the study exhibits a high level of usefulness and acceptability.

Table 1. KMO and Barlett's test of sphericity of initial iteration

KMO and Bartlett's test		Initial iteration	Final iteration
KMO measure of sampling adequacy		.933	.931
Bartlett's test of sphericity	Approx. chi-square	5518.492	5518.492
	df	861	861
	Sig.	.000	.000

Table 2 provides valuable insights into the identified factors and their associated observed variables. Examining absolute loadings greater than 0.4, it is evident that items 1-16 load significantly onto factor 1, items 17-31 load strongly onto factor 2, items 32-36 load onto factor 3, and items 37-42 load onto factor 4. Upon closer examination of the table, it becomes clear that five distinct factors have been obtained, each comprising a specific number of variables. Factor 1 encompasses 16 variables, factor 2 includes 15 variables, factor 3 consists of 5 variables, and factor 4 comprises 6 variables.

Factor 1, composed of 16 items, is characterized by statements about difficulties in various mathematical concepts and procedures. These include challenges with finding the least common denominator of rational expressions, solving equations, converting percentages to decimals, and understanding limits. This factor is labeled "lack of procedural and conceptual knowledge." Factor 2 comprises 15 items reflecting misconceptions related to integral calculations, domain determination, absolute values, logic statements, limits, and probabilistic beliefs. This factor is named "poor mathematical abstraction," signifying difficulties in abstracting and generalizing mathematical concepts. Factor 3 consists of 5 variables, indicating issues related to graph sketching, mathematical induction, probability problem-solving, general mathematical analysis, and neglecting points of discontinuity in integration. This factor is termed "internal barrier" and suggests internal challenges in understanding and processing mathematical problems. Factor 4 comprises 6 variables encompassing misconceptions such as interchanging formulas, forgetting solution steps, reliance on misleading examples, missing critical keywords, and difficulties understanding graphs' behavior with logarithms and exponential functions. This factor is labeled "cognitive conflict," indicating conflicts in cognitive structures and problem-solving approaches.

The findings align with previous research, highlighting the significance of conceptual understanding and procedural knowledge in mathematics [32]. Additionally, they reinforce the notion that mathematical misconceptions often result from challenges in computational skills and the understanding of mathematical components, algorithms, and definitions [33]. Cognitive conflict, as observed in factor 4, exemplifies the differences individuals encounter between their methods of describing mathematical concepts and those used by others [34].

4.2. Reliability of the developed questionnaire on mathematical misconceptions

Table 3 shows the factors of mathematical misconceptions and their Cronbach's Alpha score. Lack of procedural and conceptual knowledge construct has the Cronbach's Alpha Score of 0.909 which indicates a very reliable level of reliability (α>0.80-1.00) [35]. Similarly, the second identified construct, poor mathematical abstraction, has a score of 0.884, making the same range of reliability measure as the first factor. This implied that the items included in factors 1 and 2 are consistent, making the component reliable. The third and fourth components, which are depicted as internal barrier and cognitive conflict, have Cronbach's Alpha scores of 0.775 and 0.736, respectively. Both of which indicated an acceptable internal consistency (α>0.70) [35]. This means that all the items included in factors 3 and 4 are consistent, making the dimension very reliable as well.

Table 2. Rotated component matrix of self-reflected mathematical misconception scale

Item No.	Components			
	Factor 1	Factor 2	Factor 3	Factor 4
1	0.731			
2	0.637			
3	0.615			
4	0.518			
5	0.525			
6	0.699			
7	0.513			
8	0.532			
9	0.474			
10	0.477			
11	0.555			
12	0.515			
13	0.466			
14	0.476			
15	0.423			
16	0.565			
17		0.577		
18		0.574		
19		0.463		
20		0.518		
21		0.520		
22		0.559		
23		0.598		
24		0.575		
25		0.651		
26		0.500		
27		0.457		
28		0.456		
29		0.618		
30		0.461		
31		0.455		
32			0.610	
33			0.505	
34			0.665	
35			0.617	
36			0.524	
37				0.572
38				0.507
39				0.550
40				0.621
41				0.460
42				0.575

Table 3. Factors and Cronbach's Alpha score of the self-reflected mathematical misconception scale

Factor	Reliability statistics	
	Cronbach's Alpha	N of items
Lack of procedural and conceptual knowledge	0.909	16
Poor mathematical abstraction	0.884	15
Internal barrier	0.775	5
Cognitive conflict	0.736	6

4.3. Adequacy of the scale's internal consistency

To assess the indicators' validity and reliability as a measurement tool, a confirmatory analysis was conducted as shown in the path analysis in Figure 1. The path analysis using CFA is depicted through schematic diagrams where circular shapes symbolize latent variables, and square shapes represent observed [36]. The double-headed arrows denote the covariance between the four latent factors, while the single-headed arrows indicate the anticipated direction of influence [37]. Standardized residual covariance measures how well the observed data matches the expected data in covariance structure models, with higher absolute values indicating a poorer fit [38]. The factor loadings, which are standardized values representing the relationship between items and their associated factors, ranged from 0.40 to 0.71. These values fall within the acceptable range, with all exceeding 0.10, as defined by McNeish *et al*. [39].

Specifically, the "lack of procedural and conceptual knowledge" construct consisted of 12 items with factor loadings ranging from 0.58 to 0.71. The "poor mathematical abstraction" construct comprised 13 items with factor loadings ranging from 0.52 to 0.63. The "internal barrier" construct included eight items with factor loadings between 0.53 and 0.63. Finally, the "cognitive conflict" construct encompassed eight items with factor loadings ranging from 0.40 to 0.62. These factor loadings indicate the strength of the relationship between each item and its corresponding factor.

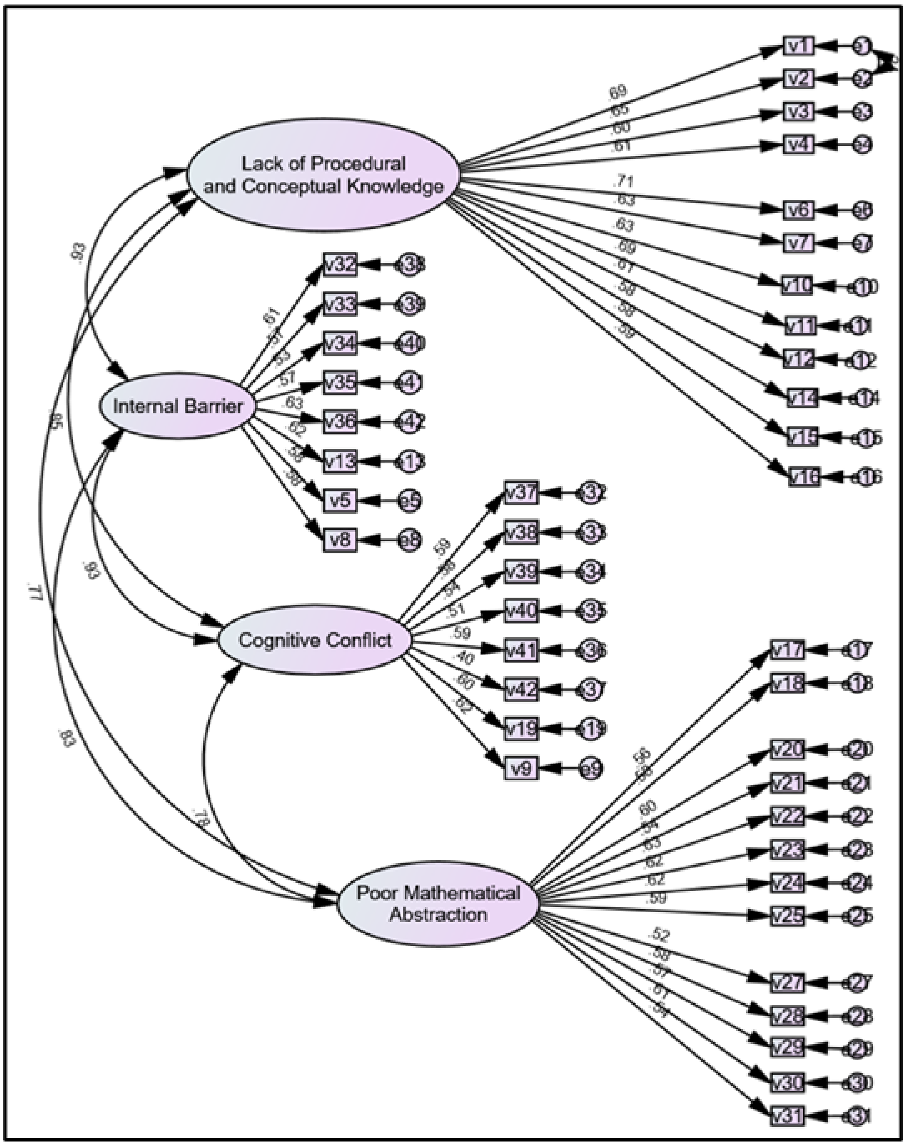

Figure 1. Path analysis in CFA

Table 4 shows the significance of Pearson's chi-square adjustment test (χ^2) which yields a value of 1503.356 with p=0.000, indicating statistical significance. Nevertheless, it is important to note that the chi-square statistic is highly sensitive to sample size [40]. As a result, the researcher opted to utilize alternative fit indices to evaluate the model, including the comparative fit index (CFI), the tucker-lewis index (TLI), and the root mean square error of approximation (RMSEA) [41]. The CFI and TLI values range between 0 and 1, with values closer to 1 being considered more indicative of a good fit [42]. Additionally, a value of RMSEA less than 0.06 is typically regarded as indicative of a well-fitting model [43].

Table 4. Pearson's Chi-square adjustment test result of the self-reflected mathematical misconception scale

Minimum achieved	
Chi-square	1503.356
Probability level	0.000

When Tables 5 and 6 were examined, the model's validity is evident. The CFI yielded a value of 0.847, while the TLI registered at 0.838 as shown in Table 5. Both values fall within the accepted range of 0 to 1, affirming the model's validity. Furthermore, the RMSEA recorded a value of 0.055 as shown in Table 6, signifying a strong fit to the hypothetical model.

Table 5. TLI and CFI of the self-reflected mathematical misconception scale

Model	TLI	CFI
Default model	0.838	0.847
Saturated model	1	1
Independence model	0	0

Table 6. RMSEA test of the self-reflected mathematical misconception scale

Model	Square error of approximation
Default model	0.055
Independence model	0.137

Table 7 shows the comparison of the reliability statistics before and after CFA. After completing the CFA and adjusting the items, the instrument was confirmed with a total of 41 items. This post-analysis refinement resulted in slight changes in Cronbach's Alpha score for factor 1, while factors 2, 3, and 4 exhibited higher internal consistency than their initial configurations. This suggests that moving items to their appropriate factors, where they exhibit internal consistency, improved the overall reliability of the instrument, achieving a more acceptable level of reliability compared to the initial 42-item version.

Table 7. Comparison of the Cronbach's Alpha score before and after CFA

Factor	Reliability statistics			
	Before		After	
	Cronbach's Alpha	N of items	Cronbach's Alpha	N of items
Lack of procedural and conceptual knowledge	0.909	16	0.888	12
Poor mathematical abstraction	0.884	15	0.867	13
Internal barrier	0.775	5	0.806	8
Cognitive conflict	0.736	6	0.781	8

5. DISCUSSION

While earlier studies explored the identification of most common mathematics misconceptions of students across grade levels, our study innovated by developing and validating a SMMS designed to facilitate an early and efficient detection of misconceptions, allowing for the design of targeted strategies to address them. Moreover, the scale is also intended to increase students' awareness of their own misconceptions through self-reflection. Through a systematic process of EFA and CFA, we successfully identified the underying constructs of mathematical misconceptions and validated a 41-item SMMS. This scale is composed of four valid factors namely: lack of procedural and conceptual knowledge (12 items), poor mathematical abstraction (13 items), internal barrier (8 items), and cognitive conflict (8 items). The developed questionnaire demonstrated high internal consistency and reliability across all four factors. This indicates that the questionnaire items consistently measure the intended constructs, making it a good tool for assessing mathematical misconceptions.

In a more detailed context, the items under "lack of procedural and conceptual knowledge" factor implies a specific dimension of students' mathematical understanding that contributes to their difficulties in learning and applying mathematical concepts. This factor encompasses two key aspects: procedural knowledge and conceptual knowledge. Procedural knowledge refers to a student's ability to execute mathematical procedures, such as carrying out computations, following algorithms, and applying specific methods to solve mathematical problems [44]. Our findings revealed that students lack the level of proficiency in computational skills and struggle with the practical aspects of performing mathematical operations. They have problems in accurately and efficiently executing mathematical procedures, which may lead to errors and misconceptions [11], [45]. On the other hand, conceptual misconceptions suggests that students not only struggle with procedural aspects but also face challenges in understanding the underlying concepts and principles of mathematics. Conceptual knowledge involves understanding the foundational ideas that connect different mathematical concepts [9]–[10], [46]. Our findings indicates that difficulties in this area may lead to misconceptions where students may have an erroneous understanding of the relationships between mathematical concepts. This misconception factor is similar with the findings of Syahrir *et al*. [47] which revealed that conceptual and procedural misconceptions are common difficulties of students across levels and across mathematics subjects.

As for the items under "poor mathematical abstraction" factor, this pertains to the difficulty in constructing the concepts in mathematics. Mathematical abstraction refers to the process of understanding and manipulating mathematical concepts beyond concrete examples [48]. Our result shows that many students struggle with this cognitive process, leading to a higher likelihood of misconceptions. It further implies that students who face challenges in abstracting mathematical ideas may find it difficult to interpret complex concepts accurately, potentially resulting in misconceptions. This finding is similar with the findings of Kadarisma *et al.* [49] which suggests a correlation between students' misconceptions in mathematics and their ability to engage in mathematical abstractions. In this context, we posit that the level of students' mathematical abstraction ability plays an important role in the occurrence of misconceptions. Our findings further imply that as students enhance their skills in mathematical abstraction, they become less prone to misconceptions and are better equipped to grasp the underlying mathematics principles. Moreover, educators and policymakers can use these insights to design instructional strategies that focus on enhancing students' mathematical abstraction abilities.

On the other hand, "internal barriers" factor were items that are associated with influences exerted by individuals encompassing their interests and perceptions of the difficulty associated with mathematics. These influences pertain to the internal aspects of individuals, including their personal interests and perceptions of the difficulty associated with mathematics or other subjects. Moreover, this identified misconception implies that individuals may develop certain beliefs about the complexity or difficulty of mathematics. These perceptions can act as psychological barriers, affecting their confidence and willingness to tackle mathematical problems. Sharif [50], negative perceptions may lead to avoidance behaviors or a reluctance to put effort in understanding mathematical concepts. Our findings suggest that building a positive attitude towards mathematics, promoting interest, and addressing misconceptions about the difficulty of the subject can help mitigate some of these internal barriers and enhance students' overall learning experience in mathematics.

Lastly, items under "cognitive conflict" factor pertains to a state of cognitive incongruity, where there is a lack of harmony between an individual's pre-existing cognitive structures and the incoming external information [51]. Our finding reveals that this misalignment leads to a state of uncertainty, doubt, confusion, contradictions, and conflict leading to misconceptions. Mathematical concepts often build upon one another, and when there is a misalignment between a student's prior knowledge and new information, misconceptions can easily emerge. These misconceptions, if not addressed, can persist and hinder further learning.

This study has provided a valuable tool for assessing and addressing mathematical misconceptions while shedding light on their underlying factors. The analysis of these factors explains the multifaceted nature of mathematical misconceptions among senior high school STEM students in the Philippines, providing valuable insights for educational interventions and curriculum design aimed at addressing these misconceptions and enhancing mathematical learning.

6. CONCLUSION

The developed and validated scale for identifying misconceptions in senior high school mathematics is a valuable tool that can provide valuable insights into areas that require focused attention to improve students' mathematical proficiency. The study offers several key recommendations: First, educators should harness the validated questionnaire as a valuable tool to assess and address students' mathematical misconceptions, identifying areas where students struggle most and enabling tailored instructional strategies. Second, educators can formulate targeted intervention strategies informed by the identified factors, potentially employing additional practice and conceptual teaching methods for those lacking procedural and conceptual knowledge and integrating conflict resolution strategies for students experiencing cognitive conflicts. Third, curriculum developers should utilize the study's findings to inform the development of more effective and comprehensive curricula, addressing common mathematical misconceptions from the outset. Finally, further research is encouraged to deepen our understanding of the causes and remediation of mathematical misconceptions, potentially through longitudinal studies tracking students' progress in overcoming these challenges.

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AUTHOR CONTRIBUTIONS STATEMENT

For clarity and comprehensive attribution, the following table lists each author's contributions according to the CRediT taxonomy.

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Karen A. Quinio	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	
Polemer M. Cuarto	✓	✓		✓	✓	✓		✓	✓	✓	✓	✓		

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

The authors declare that they have no conflicts of interest related to this research and no financial interests or relationships with organizations that might have influenced the outcome of this research.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author, [PMC], upon reasonable request.

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