Journal of Education and Learning (EduLearn)
Vol. 13, No. 2, May 2019, pp. 201~211
ISSN: 2089-9823 DOI: 10.11591/edulearn.v13i2.8996

Mathematical modeling for learning algebraic operation

Diah Dwi Santri, Yusuf Hartono, Somakim
Department of Education Mathematics, Sriwijaya University, Indonesia

Article Info

Article history:

Received Jun 1, 2018
Revised Mar 17, 2019
Accepted Apr 20, 2019

Keywords:

Algebraic operations
Desain research
Mathematical modeling

ABSTRACT

This study aims to produce a learning trajectory using the mathematical modeling in helping students to understand the concept of algebraic operations. Therefore, the design research was chosen to meet the research aims and to give in formulating and developing local instructional theory in learning algebraic operations.Learning trajectory designed in the early phases and tested on 34 seven-grade students in SMP N 10 Palembang. Data collection was conducted through observation by recording the learning process that occured in the classroom and students' group work was evidenced by video and photos. Data was analyzed qualitatively by describing actual learning which happened in pilot experiment and teaching experiment. There are 3 learning activities in the design of this study. These 3 activities are designed based on the steps of the Mathematical Modeling, activity 1 meaning of algebraic expressions, activity 2 addition of algebraic and activity 3 subtraction of algebraic. Based on the result, it can be concluded that activity which has been designed can help the students in learning algebraic operations using mathematical modeling. Used mathematical modeling can help student solve the problems and understand concept are structured using the assumptions and model start they design so gradually developed into formal mathematics.

Copyright © 2019 Institute of Advanced Engineering and Science.
All rights reserved.

Corresponding Author:

Yusuf Hartono,
Departement of Education Mathematics,
Sriwijaya University,
Jalan Srijaya Negara, Palembang, Indonesia.
Email: y_hartono@yahoo.com

1. INTRODUCTION

Algebra is one of mathematical competencies that is essential to be taught to and to be understood by students of elementary school to high school level, since the skills of operating and understanding algebra symbols, namely variables, constants, terms, factors, and the operations, are the foundations to study the next step of algebra [1].

However, in real world setting according to Suswiyati [2] algebra topic is still hard to understand for students, due to the presentation of algebra forms in algebraic equations that is still abstract and difficult to comprehend. This is also supported by the research of Meny [3, 4] which stated that students' difficulties in learning algebra is about basic understanding of algebraic forms and algebraic operations.

The difficulties students faced are related to the learning style which is being applied by the teacher. This parallels to the statement of [5] that

> *"…an initial segment where the previous day's work is corrected. Next, the teacher presents new material, often working one or two new problems followed by a few students working similar problems at the chalkboard. The final segment involves students working on an assignment for the following day."*

This means that learning activities consist of three main segments including (1) check homework of the previous day, (2) present new topics to the students, and then (3) students do tasks. The remaining unfinished tasks will be continued as homeworks.

Those activities allow no room for students to create and to improve their creativity, as required by bloom taxonomy which had been revised on the highest cognitive domain CREATE [6] that mathematics is the human life activity that drives students to do actions such as experiments and investigations which lead to verification of students-made conjecture and will to do mathematical investigation and exploration in order to bring out their creative attitude.

Consequently there is the need for a mathematical learning to facilitate students in making and creating mathematical modeling which they can find by themselves. Learning algebra with mathematical modeling becomes the focus of this research.

The International Conference On The Teaching Of Mathematical Modelling And Application (ICTMAs) and The International Congresses on Mathematical Education (ICMEs) Blum [7, 8] explains that modeling is a aktivits who played an important role and significant in all levels of education and learning stages, both primary, secondary levels of education, to higher education. According to [9] mathematical modelling can be used as strategic pedagogy can complement the relationship between real applications and utilization of resources, as it provides an environment favourable for the students to understand the problem, collect their own data (the assumption), and participated in the investigation, analysis, discussion and reflection.

Several research have also obtained maximum results in learning using mathematical modeling, one of which is the research of Selvia, Darmawijoyo, and Yusuf [10] which stated that mathematical modeling application can increase mathematical problem-solving skill of students. Meanwhile, according to the research of Turmudi, Hidayat, Prabawanto, and Aljupri [11] mathematical modeling increases communication in mathematics, competencies in strategic mathematics, skils in problem-solving, and adaptive reasoning of students. The results of those previous studies suggest that the application of mathematical modeling holds a great role in algebra or other topics.

The process of modelling stated in Ang Keng Cheng [12-14] consists of construct and structurize, make assumptions, formulate equations, solve equations, interpret the solutions and validate, and finalize Because of the discussions above, researcher carried out this research with the aim of developing a theory of learning to assist students in understanding the concepts of algebraic operations using mathematical modeling..

2. RESEARCH METHOD

In this research, the method used is design research which produces a learning trajectory in algebraic operations learning using mathematical modeling. Design research aims to develop local instructional theory based on existing theory (theory-driven) and empirical experiment (empirically-based) through cooperation between researcher and teacher in order to improve research relevancy with educational policy and practice [15]. Design research has 3 steps which are: preparing for the experiment, the design experiment and the retrospective analysis [16-18] with phases as follows:

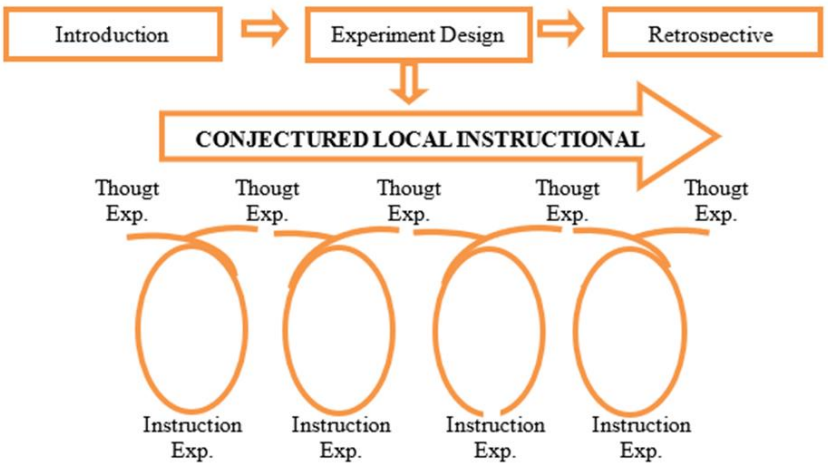


Figure 1. Design research phases [17]

3. RESULTS AND ANALYSIS

This study was designed to produce a learning trajectory in the learning material algebraic operations using mathematical modeling. This research underwent through two of the three phases of design research, namely preliminary design and the design experiment, conducted in the teaching experiment.

3.1. Preliminary design

At the phase of preliminary design, researcher examined the literature on material algebraic operations, mathematical modeling, and design research that was used as research methods. After the researcher designed the initial learning, the learning device that will be used in the experiment phase was obtained. Before researcher conducted experimental phase, researcher discussed in advance with the math teacher who became the model teacher. This is done because the teacher is more familiar with the characteristics of students who will be the subject of study. During the discussions, researcher and the teacher discuss the learning device, and if it is not in accordance with the research subjects, it will be revised. Among them are: Hypothetical Learning Trajectory (HLT), student activity sheet, lesson plans, teacher's guide, pre-test and post-test questions, as well as the observation sheet.

3.2. Teaching experiment

This phase involved 34 seventh graders of SMP 10 Palembang, Mrs. Marhamah Fajriyah, M.Pd, the homeroom teacher acted as a model teacher in this step. In the learning implementation, the teacher divided students into 6 groups of 5 students (group 1-6). The group division was based on homogenous skills for every group and heterogenous skills among every group, where each consists of high, middle and low-skilled students. In the implementation step of this teaching experiment, the researcher acted as an observer by witnessing student strategies in solving the given problems using mathematical modeling. The learning process was started by giving pre-test, three learning activities and then continued by giving post-test. Pre-test to know the students knowledge about the material to be studied, and post test to know the knowledge the students after using mathematical modeling in solve the problem of algebraic operations.

Student start the first activity is to find the meaning of algebraic expressions using problems about packaging oranges that will be sent using fruit baskets and boxes. From this problem, the researcher hoped that the students may further understand algebraic forms and understand the elements of algebraic, such as variable, coefficient, and constant. The first step in solve the problem, based on Ang [19] students should write the information contained on given problem. The answers of the students in step construct and structurize are shown on Figure 2.

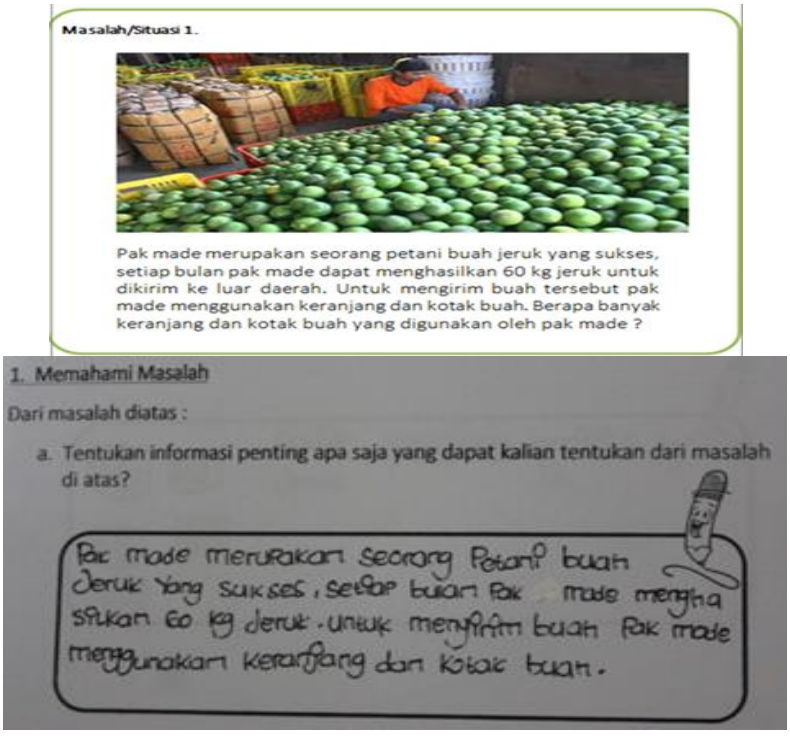


Figure 2. The answers of the students in step construct and structurize

Figure 2 show that the student write the information about the problem , the student write information about the problems is mr.made have 60 kg oranges and he using fruits basket and box. Step 2 is make assumptions, in this step Every group makes different guesses based on the given problem about the amount of baskets and boxes that will be used to pack the oranges, thus various kinds of algebra will be obtained from the given problem. In figure 2 is one of the assumptions of students.

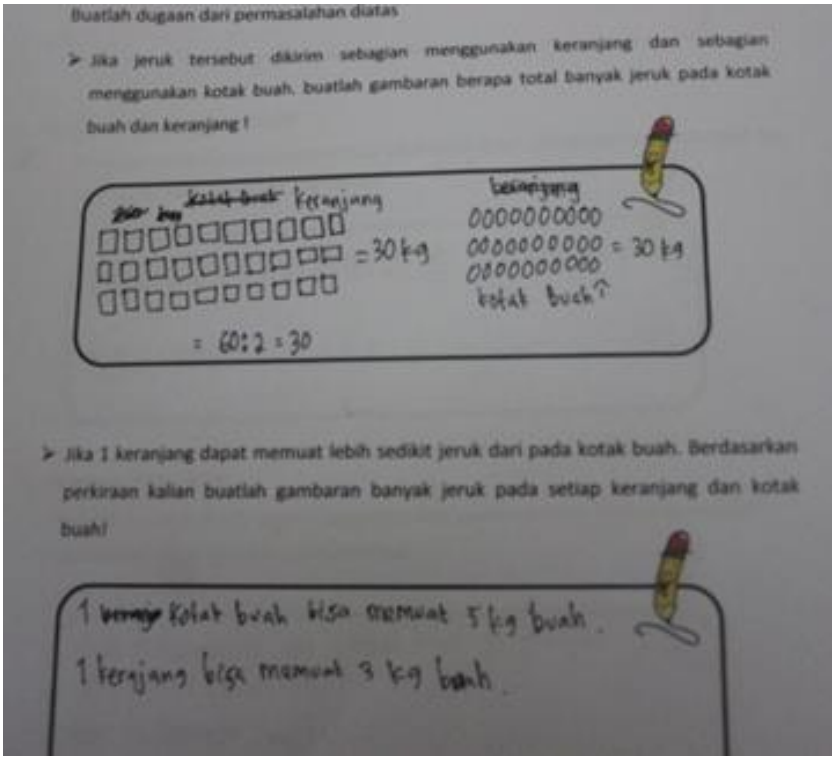


Figure 3. Assumptions of the students

Figure 3 shows that the students are able to make guesses from the given questions about how many kilograms a basket and a box may contain. The following transcript is the conversation with the student whose answers are on Figure 1.

T: Dina, how many oranges does Mr. Made have?
S: 60 kg of oranges
T: If halves are sent using fruit baskets and boxes?
S: Therefore, 60 divided by 2, equals 30
T: How will the 30 be sent?
S: The 30 will be sent using fruit baskets.
T: Oh, then the other 30?
S: They will be sent using fruit boxes.
T: If each basket can contain less than fruit box, how many kilograms of oranges can fit in the basket?
S: 1 basket can contain 3 kg of orange
T: How about the fruit box?
S: Since the fruit box can contain more orange, thus 1 fruit box can contain 5 kg.

Conversation Transcript 1. The student's answers about assumptions

After making guesses, the next step is about modeling. Students are asked to make mathematical equations and solutions by initially symbolizing fruit basket and box with the symbols they like. The purpose is to make the students know that fruit basket and box are variables, while the amount of each is coefficient. Figure 4 is the students' answers on the step of making mathematical equations and solutions.

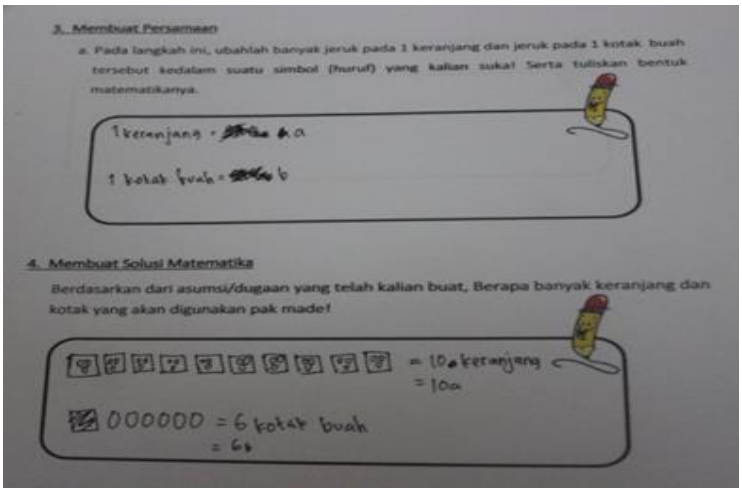


Figure 4. The student's answer of formulate equations and solve equations

Figure 4 shows that the students' answers have already led to algebraic forms. They symbolize and make solutions based on the given questions. The following transcript is a conversation with the student whose answers are on Figure 4.

T: Based on Dina's opinion, what is the answer for the problem 3?
S: 1 basket is supposed as a, 1 fruit box is supposed as b.
T: Why is that?
S: Because it asked for the symbols of 1 fruit basket and 1 fruit box that contain orange.
T: So Dina's answer is?
S: 1 basket = a, and 1 fruit box = b
T: Ok then, about the number 4, what is the mathematical solution?
S: So all the oranges to be sent using baskets are 30 kg, 1 basket can only contain 3 kg, therefore there are 10 baskets that can be used.
T: So there are 10 baskets, if 1 fruit basket contains 3 kg, what about the fruit boxes?
S: Since 1 fruit box can contain 5 kg, therefore there are 6 fruit boxes that can be used.
T: So, if the fruit box and the basket are symbolized as the number 3, what will the answer for the number 4 be?
S: 1 basket was supposed as a, since there are 10 baskets, it becomes 10a. Meanwhile the fruit box was supposed as b therefore there are 6b.

Conversation Transcript 2. The student's answers on student activity sheet 1

The next modeling step is making conclusion from the guesses and assumptions that have been made by the students. The following Figure 4 is the conclusion made by the students and the conclusion of the algebraic forms they have just learned.

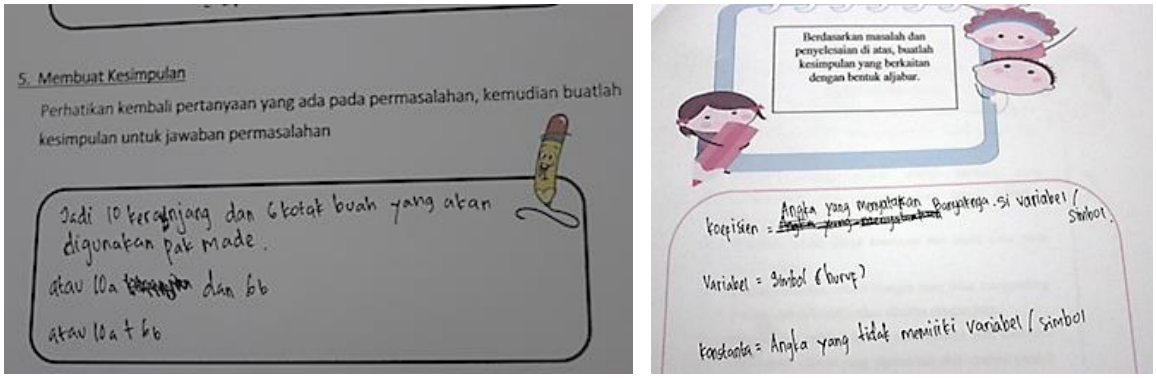


Figure 5. The student's answer on interpret solutions and validate, and finalize

The following transcript is a conversation with the student whose answers are on Figure 5.

T: So what is the conclusion of the given problem?

S: So there are 10 baskets and 6 fruit boxes that are used by Mr. Made. Or these can be written as 10a + 6b

T: So in Dina's opinion what are variable, coefficient, and constant?

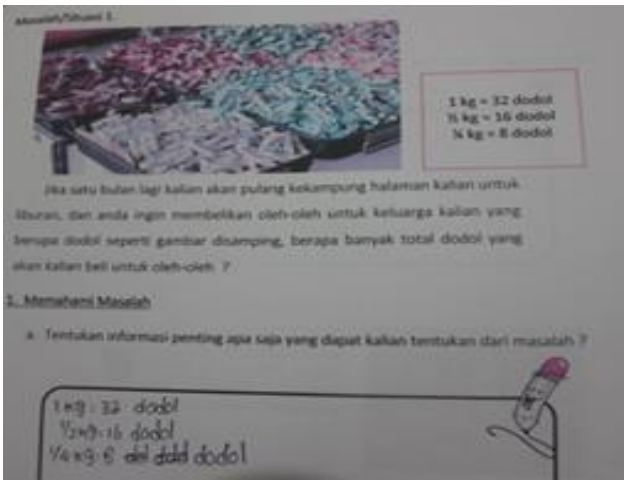
S: Variable is symbol or letter, coefficient is number that states the amount of variable or symbol, while constant is number that doesn't have variable or symbol.

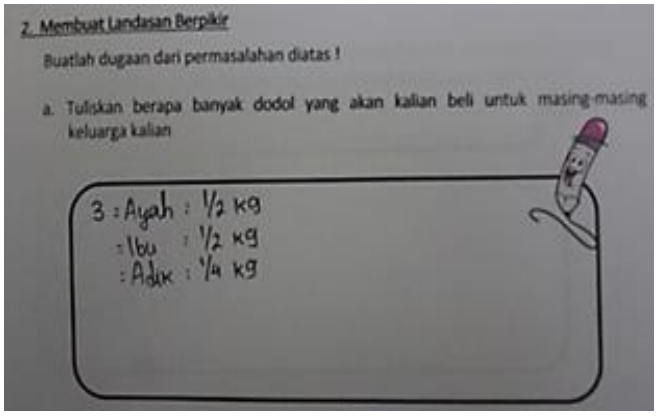
T: Thank you Dina.

Conversation Transcript 3. The student's answers on student activity sheet 1

In the defining the problem, students are able to mention some information. However, students do not use some of the information mentioned about the terms of purchase of dodol 1 kg = 32 dodol, ½ kg of dodol and 16 = ¼ kg = 8 lunkhead. In other words, the student group 1 has not been able to do the entire process of defining the problem. Biccard said that the understanding/defining the means to understand the outline of the information required [20]. But students can classify models of assumptions that have been made as shown in Figure 2, students can make assumptions and models of oranges that are sent using the Orange basket and sent using a fruit box (making assumptions). This causes the students can create equations (defining the variables/equations) of assumptions they have made from the previous phase. Biccard [20] that matematisasi/make assumptions is a structure from real situations using mathematical ideas and concepts. Students can make conclusions with regard to the form of the algebra (getting the solutions). Students can show a formula that connects between the known information.

In student activity sheet 2, students will learn about algebraic addition. This activity sheet consists of two learning activities based on two different problems. The students are required to solve an issue which is "if you want to buy *dodol* for each of your family members". They are asked to use the area model in the process of finding the sum of algebra. The activities can be seen on Figure 6.





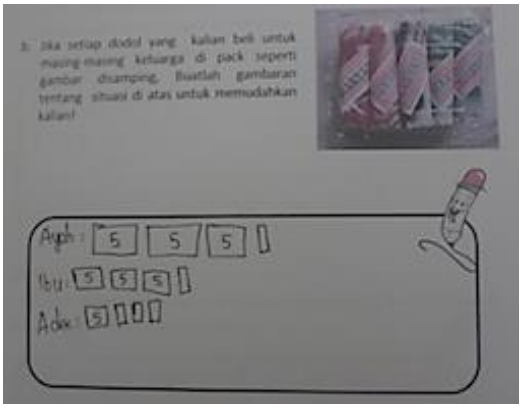


Figure 6. The student's answer in step construct and structurize, make assumptions

Figure 6 shows that students made assumptions from the problem about the *dodols* that will be bought for the family. They wrote the amount of the *dodols* which are to be given to the family members and created an area model of the *dodols* which are to be sold in packages. The following transcript is a conversation with the student whose answers are on Figure 5.

T: According Rafli's, what about the given problem?
S: From the question it is given that 1 kg = 32 dodols, kg = 16 dodol, and ¼ kg = 8 dodol
T: So, how many dodol does Rafli's want to buy for each of the family members?
S: Father ½ kg, mom ½ kg and little brother ¼ kg.
T: Oh, so if the dodol are packed, how will they be?
S: If 1 pack can only contain 5 dodol, therefore according to the created model it is obtained 3 packs and 1 dodol that is not packed.
T: About little brother that will be given ¼ kg, how?
S: So little brother gets 1 pack and 3 dodol that are not packed.

Conversations Transcript 4. The student's answer on student activity sheet 2.

In the step of making equations, students are asked to convert the packed *dodols* into a symbol and write it down in its algebraic form. Therefore the answers obtained are shown on Figure 7 below.

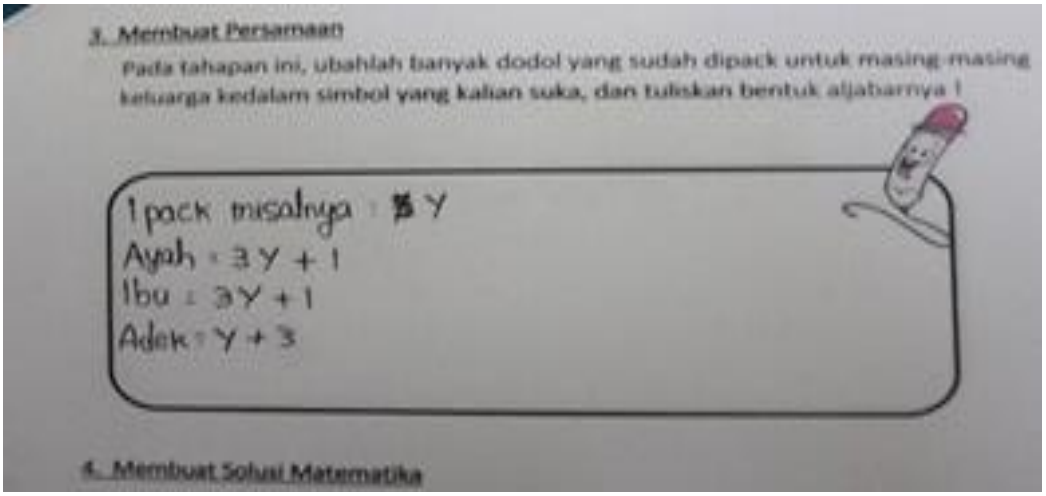


Figure 7. The student's answer of formulate equations

Figure 7 shows that the student makes equations by symbolizing the packed *dodols* first, and then its algebraic form. The following transcript is a conversation with the student whose answers are on Figure 7.

Conversations Transcript 5. The student's answer on student activity 2
T: Ok Rafli, so what is Rafli's answer for the step of making equations?
S: I suppose 1 dodol pack with the y letter symbol, then i write its algebraic form which are father: 3y + 1, mother: 3y + 1, and little brother: y + 3.
T: How were those obtained?
S: Father in the step of modeling got 3 dodol packs and 1 dodol unpacked, simce the packed dodols are supposed by me as y therefore becomes 3y + 1
T: 1 refers to?
S: The unpacked dodol.

In the step of making solutions, students are asked to think about how to add in the algebraic forms that have been obtained before. Hence the number of dodol they have to buy for the family will be obtained. After that, they are required to make conclusion and to check their answer according to the assumptions he have written.

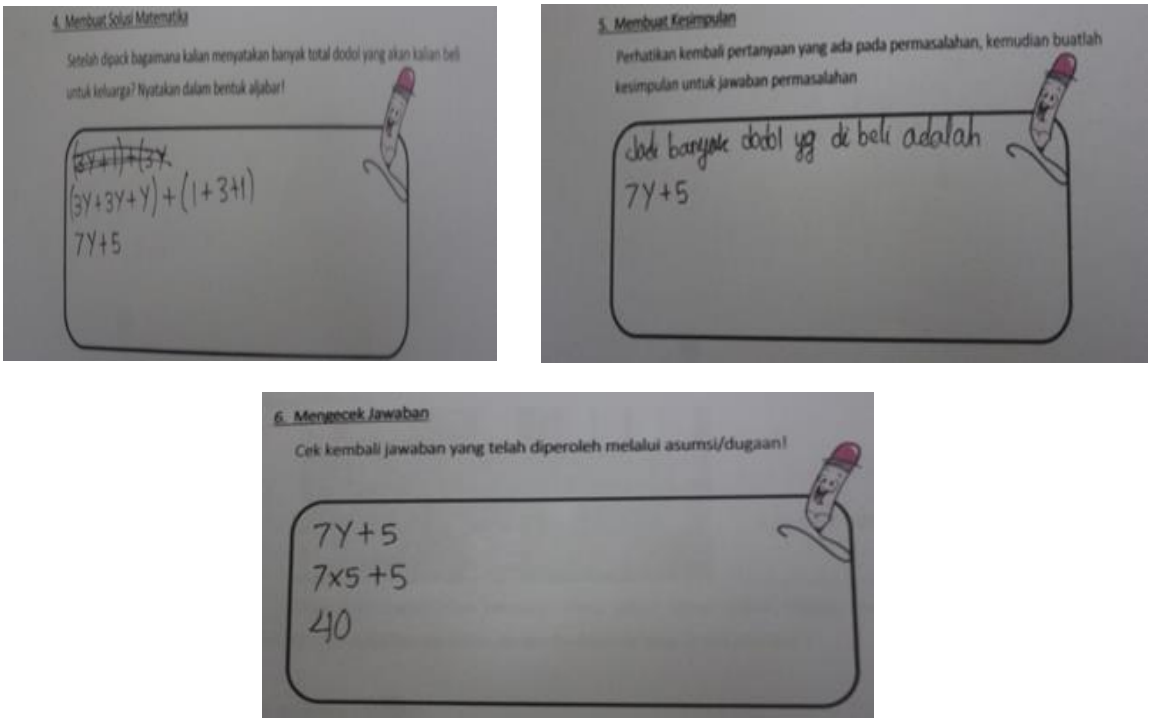


Figure 8. The student's answer on student activity sheet 2

The following transcript is a conversation with the student whose answers are on Figure 8.

T: How will Rafli state the total amount of dodol that are bought in algebraic form?
S: So by adding the same symbol and constant with constant.
T: So what is the result?
S: So (3y+3y+y) + (1+3+1), therefore it is obtained 7y + 5.
T: Then, what is the conclusion Rafli makes?
S: So the amount of dodol that is bought is 7y + 5.
T: In the step of re-checking, how?
S: The amount that is bought was 7y + 5, we replace the former y with the containing of the dodol pack which is 5, therefore 7x5 + 5 = 40. Same with the amount of dodol that is bought 16+16+8=40.

Conversations Transcript 6. The student's answer on student activity 2

After working on the given problems, students are asked to provide conclusions related to the algebraic operations. Figure 9 shows the students's conclusions.

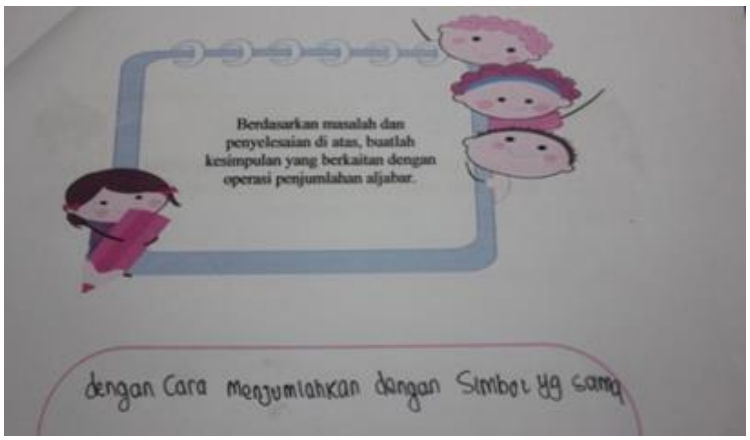


Figure 9. Activities of students conclude about algebraic addition

In student activity 3, students will learn about algebraic substraction. Students are required to solve an issue where Lisa bought a chocolate and then ate it with her friends. In this activity, students are asked to use area model in the process of finding the result of algebraic substraction. This activity is shown by the following Figure 10.

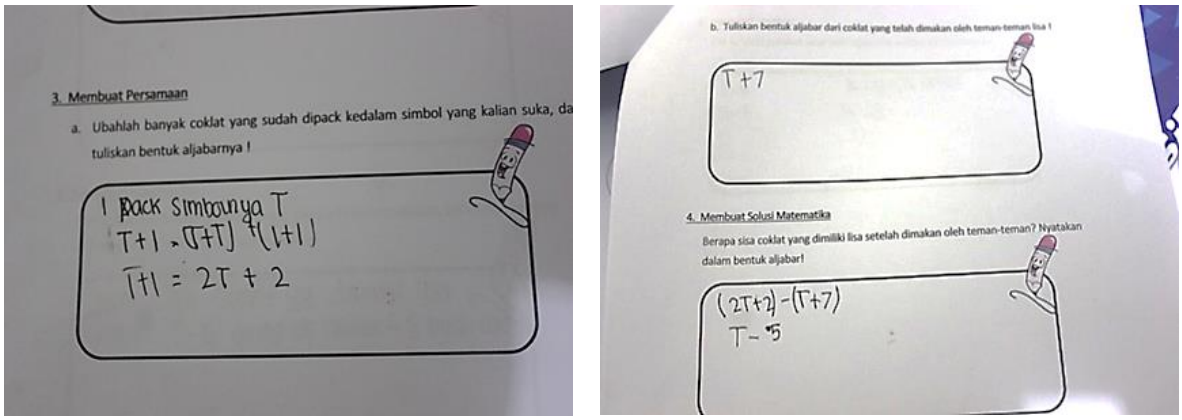


Figure 10. Activities of students on student activity sheet 3

Figure 10 shows that students conducted algebraic subtraction using the steps of mathematical modeling. They made a model from the given problem and subtract the algebraic form of the model. The following transcript is a conversation with the student whose answers are on Figure 9.

T: Where did you get 2t + 2?
S: We made assumptions that 1 kg of chocolate contains 17 pieces of chocolate, therefore if Lisa bought 2 kg of chocolate there will be 34 pieces of chocolate.
T: After getting 34, what did you do?
S: We made model, if 1 box of chocolate contains 16 therefore there are 2 boxes of chocolate and 2 chocolate that can't be packed.
T: So there are 2 boxes of chocolate and 2 chocolates that are not packed?
S: Yes, we supposed the box of chocolate as t, therefore it is obtained 2t + 2
T: Is 2t + 2 the remaining chocolate of Lisa?
S: No, 2t + 2 is the amount of chocolate Lisa had, the amount eaten by Lisa's friends are 23 if those are modelled it will be obtained 1 box of chocolate and 7 chocolate that are not packed.
T: So what is that 1 box supposed as? How is its algebraic form?
S: That 1 box is supposed as t, therefore the algebraic form of the eaten chocolate is t+7.
T: So, how many of Lisa's remaining chocolate are there?
S: Just subtract (2t + 2) - (t + 7) therefore the remaining are t - 5 or 11 pieces, since t = 16.

After the students work on the third LAS, students are asked to work on the final phase of the test (post-test) individually. The assessment of post-test showed different results with a pre-test. The students were not confused in adding and subtracting algebraic forms. During the discussion in teaching experiment phase, the students said that through using mathematical modeling, the learning became more interesting and creative. Students can also understand instantly that the algebraic addition and subtraction cannot be added directly, but must be synchronized first with symbols. This is consinstent with the theory which says that one model area can be used in learning algebra.

4. CONCLUSION

Based on the results and discussion that has been described previously, it can be concluded that application of HLT in this study has been a learning trajectory that could help students understand and resolve problems algebraic operation from informally intuitive to the formal problem-solving by using the mathematical modeling. The things in the trajectory of learning in this study were helping students as follows: Experience in meaningful learning and creative activities by student can determine their assumption/conjecture in answering a problem so the answer that owned each student is more diverse, and

Through mathematical modeling as one of the alternatives of teaching mathematics to students, can apparently increase the skills of communication, adaptive reasoning, mathematical strategies, as well as problem-solving. Therefore it is not false to apply mathematical modeling as one of the strategies to teach mathematics in high school

ACKNOWLEDGEMENT

Thanks to teachers of Junior High School 10 of Palembang who gave me the opportunity to conduct research in all schools.

REFERENCES

[1] Wardhani, *Contextual problems introduce algebraic forms in middle school* (in Bahasa), Yogyakarta: Depdiknas, 2014, pp. 5-10.
[2] Suswiyati, "Accurate moves to increase students' creativity in completing the algebraic form factorization questions using rectangular pieces of paper class VIII C SMP N 1 Paliyan Gunungkidul 2011/2012 Academic Year (in Bahasa)," *Seminar Nasional Matematika dan Pendidikan Matematika dengan tema "Matematika dan Pendidikan Karakter dalam Pembelajaran"*,Yogyakarta, pp. 458-459, 2011.
[3] Meny. R, "Difficulty analysis of class VII students on algebraic material at MTsN 2 Banda Aceh (in Bahasa)," Skripsi, Universitas Syiah Kuala Aceh, Banda Aceh, 2016, pp. 55-56.
[4] Rahayu D, P, "Student mistake analysis in Algebra calculating operating materials (in Bahasa)," *Kompasiana*, 2017.
[5] Romberg. T. A and Kaput. J. J, *Mathematic worth*, New Jersey: Larence Elbraum, Inc, 1999, ch.1.
[6] Anderson and Krathwohl, *A taxonomy for learning,teaching,and assesing; A revision of bloom's taxonomy of education objective*, New York: Addison Wesley Lonman Inc, 2001, pp. 212-218.
[7] Blum, W, "ICMI Study 14: Applications and Modelling in Mathematics Education – Discussion Document", *Educational Studies in Mathematics*, vol. (3/4), pp. 262-280, 2002.
[8] Blum, W., Galbraith, P.L., Hans-Wolfgang, H., and Niss, M, "Study ICMI 14th: Modelling and application in mathematics education," *Educational Studies in Mathematics*, 2007.
[9] Ferreira, D, H, L., and Jacobini, O, R. "Mathematical Modeling: From Classroom To The Real World," In *Mathematical Applications and Modelling in the Teaching and Learning of Mathematics*, Mexico, 2008.
[10] Selvia. F. R, Darmawijoyo and Yusuf. M, "The application of learning mathematical modeling using a constructivism approach to problem solving ability for class VIII Middle School Students (in Bahasa)," *Jurnal Pendidikan Matematika*, vol. 3, pp. 66-74, 2014.
[11] Turmudi, Hidayat. A. S, Prabawanto.S, and Aljupri, "development of mathematical learning with realistic based mathematical modeling for students (in Bahasa)," *Jurnal Pengajaran MIPA*, vol. 19 pp. 1-18, 2014
[12] Ang, K. C, "Teaching Mathematical Modelling in Singapore Schools," *The Mathematics Educator*, vol. 6, no. 1, pp 63-75, 2001
[13] Ang. K.C, "Mathematical Modelling, Technology and H3 Mathematics", *The Mathematics Educator*, vol 9, no. 2, pp. 33-47, 2006.
[14] Ang. K. C, "Mathematical Modelling and Real Life Problem Solving," *World Scientific*, pp. 159-182, 2009.
[15] Gravemeijer. K and Eerde. D.V, "Design Research as a Means for Building a Knowledge Base for Teaching in Mathematics Education," *The Elementary School Journal*, vol.109, pp. 510-524, 2009.
[16] Gravemeijer. K and Cobb. P, "Design Research From the Learning Design Perspective," In J. V. Akker, B. Bannan, A. E. Kelly, N. Nieveen, and T. Plomp, *Educational Design Research*, Netherlands: SLO-Netherlands institute for curriculum development, 2013, pp. 72-133.
[17] Gravemeijer K, "Local instruction theories as means of support for teachers in reform mathematics education," *Mathematical thinking and learning*, 2004**,** pp. 105.
[18] Bakker, A,. *Design Research in Statistic Education on Symbolizing and Computer Tools*, Amersfoort : Wilco Press, 2004.
[19] Ang. K. C, "Mathematical Modelling in Singapore Schools: A Framework for Instruction," *World Scientific*, pp. 57-72, 2015.
[20] Biccard, P, "An Investigation the Development of Mathematical Modelling Competencies of Grade 7 eaners," Unpublished Thesis, Stellenbosch University, 2010.

BIOGRAPHIES OF AUTHORS

My name is Diah Dwi Santri, S.Pd
I am student's of magister education mathematics in Sriwijaya University
My research is "Desain Soal Pemodelan Matematika Berdasarkan Teori Gaimme" in publication proceding Seminar Nasional Matematika dan Pendidikan Matematika 2017

Dr. Yusuf Hartono
Lecturer in Department of Educations Mathematics in Sriwijaya University
Dr. Yusuf Hartono does research in Statistics, Probability Theory and Number Theory and Educations. Their most recent publication is "Exploring Primary Student's Problem-Solving Ability by Doing Tasks Like Pisa's Questions".
Skill and Expertise : Mathematical Modeling.

Dr. Somakim, M.Pd
Lecturer in Department of Educations Mathematics in Sriwijaya University
Dr. Somakim does research in Learning in mathematics educations, Pisa, Curiculum, and Development of Media. Their most recent publication is "Pengembangan Media Pembelajaran Matematika Pokok Bahasan Segitiga Menggunakan Macromedia Flash untuk Siswa Kelas VII SMP".
Skill and Expertise : Curiculum, Learning