

Towards a Comprehensive Conception of Mathematical Proof

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ABSTRACT

There is overwhelming evidence that students face serious challenges in learning mathematical proof. Studies have found that students possess a superficial understanding of mathematical proof. With the aim of contributing to efforts intended to develop a comprehensive conception of mathematical proof, literature search was conducted to identify areas where research could be directed in order to increase proof understanding among students. To accomplish this goal, literature on modes of reasoning involved in proof construction, ideas on the classification of activities that constitute a proof path, and categories of proof understanding are exemplified using mathematical content drawn from Real Analysis. These exemplifications were used to illustrate the connections between modes of reasoning and levels of proof understanding. With regard to students' fragile grasp of mathematical proof this critique of literature has revealed that many previous studies have given prominence to proof validations while there is lack of crucial interplay between structural and inductive modes of reasoning during proving by students. Hence, it is suggested in this paper that current research could also focus on mechanisms that promote an analytic conceptions of mathematical proof that are comprehensive enough to allow students to engage in more robust proof constructions.

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1. INTRODUCTION

This paper discusses aspects of knowledge of content and students (KCS) which Lesseig [1] describes as knowledge of students' typical conceptions or misconceptions of mathematics. In Zimbabwe the mathematics education community faces the challenge of improving students' abilities to autonomously produce proofs of mathematical statements at all scholastic levels. Generally, students focus on reproducing proofs from lecture notes, yet mathematics learning requires far more than simply working on exercises and doing desired computations and regurgitation of routine proofs. But why do we need to pay attention to mathematical proof?

The concept of mathematical proof has a central place in the learning of mathematics because of its potential to promote justification and understanding as suggested by Olsker [2]. Mathematical proving promotes students argumentative skills which are essential for developing deep learning. However, in spite of several benefits mathematical proof has failed to permeate the curriculum at all scholastic levels as reported by Stylianides [3]. In other words, studies have shown the absence of a good understanding of proof and proving among learners.

There is overwhelming evidence that students face serious challenges in comprehending proofs. Jahnke [4] has remarked that “many school and university students and even teachers of mathematics have only superficial ideas on the nature of proof”. Yet the knowledge of teachers related to proof and proving directly influences their way of teaching proof. Further, limited knowledge of proof allows misconceptions in many students regarding proofs to persist as suggested by Uğurel *et. al.* [5]. Hence, if undergraduate mathematics education students do not master the concept of mathematical proof adequately they are less likely to develop it in their own learners in a persuasive manner. Undergraduate student teachers should therefore have a deep understanding of mathematical proof and proving. Hence, studies to determine teachers’ competences in content knowledge in mathematical proof are crucial.

Harel and Sowder [6] write that the problems that students experience with mathematical proof are a result of the manner in which the concept of mathematical proof is presented to students. Proof is usually presented as a finished product so that students sometimes lack the intellectual curiosity to wonder why a given mathematical statement is true as proposed by Stylianides [3]. The lack of intellectual curiosity stems probably from learners’ view of the role of proof as a tool needed to confirm something that is intuitively obvious and is already known to be true. An exemplification of this point is the proof of the theorem: *The square root of 2 is irrational*. Before the truth seeking activity (proving process), students are well aware of the fact that the square root of 2 is irrational. Ndemo and Mtetwa [8] suggest the source of such awareness among learners can be explained in terms of their met-befores, which precisely refer to the learners’ previous experiences with irrational numbers. In secondary school mathematics, students would have looked at topics that involve use of irrational numbers such as: *Quadratic equations with inexact roots*. Thus, the proposition: *The square root of 2 is irrational*, is perceived as something that is intuitively obvious in the sense suggested by Harel [9], hence, the lack of intellectual curiosity in the proposition’s validation. Such a viewpoint is a consequence of the manner in which mathematics is usually presented to learners—a finished product.

Following Wilkerson-Jerde and Wilensky [10] we argue that most challenges faced by students in learning a mathematical concept are related to the nature of the network of mathematical resources formed by an individual. To clarify our argument about the influence of the nature of the network of resources on one’s conception we draw on Duffin’s categorization of mathematical understanding. Duffin and Simpson [11] categorize mathematical understanding by differentiating building, having, and enacting as different components of mathematical understanding. Duffin and Simpson [11] describe building as an aspect that refer to the process of developing the connections, having denotes the state of the connections, and enacting is used to describe the process of applying the connections available to solve a problem.

From Duffin and Simpson [11] categorization we can therefore assert that learning new mathematics can be thought of as the creation of a network of mathematical resources. If the network of mathematical resources is not well coordinated, then one’s conception of the concept can be considered to be weak. Hence, students’ superficial understanding of mathematical proof is related to the students’ network of resources with respect to the concept of mathematical proof, which in turn determines their manner of understanding of the concept.

Before describing the nature of problems related to students’ fragile grasp of the notion of mathematical proof we shall first examine basic constructs that inform social science research. According to Charmaz [12] there are two fundamental notions in social science research, namely, basic social problem and basic social process. A basic social problem refers to a problematic phenomenon from the point of view of people being studied. Charmaz [12] says for something to qualify as a basic social problem, it must not be short-lived. Mathematical philosophers, mathematicians and mathematics educators have been grappling with the notion of proof for years and as such students’ difficulties with proof can count as a basic social problem. Charmaz [12] describes a basic social process, as what the participants (people being studied) essentially do in dealing with their basic social problem. In the context of this article the basic social problem is superficial understanding of mathematical proof by undergraduate mathematics student teachers as reflected through the students’ rote memorization of routine proofs and lack of intellectual curiosity and appreciation for meaning in proof constructions. In other words, there is lack of deep understanding of mathematical proof among Zimbabwean undergraduate student teachers.

Evidence of the basic social process, that is, what students do as a way of dealing with their weak command of the concept of mathematical proof includes:

- a. Students have been so inept at producing deductive arguments.
- b. The concept of mathematical proof has failed to permeate the undergraduate mathematics curriculum because of the tendency to focus on rote memorization and regurgitation of routine instructors’ notes by learners. Further, Pfeiffer [13] writes that instructors rarely engage learners in construction of novel proof tasks.
- c. Martin [13] report that students experience extreme difficulties with proof tasks to an extent that some do not even know how to begin the proving process.

- d. Harel [6] write that student teachers insist on being told the proofs rather than taking part in the proof construction process.

Students' struggles with mathematical proof articulated here have been persistent. Studies have attributed students' difficulties with mathematical proof to the abstract nature of the concept of mathematical proof as has been reported by Ndemo and Mtetwa [8]. The act of proving distinguishes mathematics from other disciplines because it involves abstraction—a process used to obtain the essence of a mathematical concept through the structural mode of reasoning as opposed to the concrete operational mode. Herlina and Batusangkar [15] say the structural mode of thought omits dependence on real world objects such as numeric examples which help one to grasp some sense of the mathematical relationship. It is therefore critical that from the onset of any study on mathematical proof and proving a more powerful definition of proving is developed.

Hence, in this section some definitions of mathematical proof are discussed. During the discussion some flaws with those definitions are examined in order to orient current research towards a direction that is likely to foster the development of a comprehensive view of mathematical proof. Such a direction, we believe, has the potential to promote thinking processes about proving. In so doing challenges faced in connection with proofs may be ameliorated.

2. CRITICAL REVIEW

Definitions of mathematical proof include;

- A socially sanctioned written product that results from mathematicians' attempts to justify whether a given conjecture is true as defined Weber and Mejia-Ramos [16].
- Mathematical proving is the process of searching for arguments used to convince a person or a community, that is, to justify the accuracy of a mathematical statement in the sense suggested Bieda [17]. Bostic [18] describes a justification or argumentation as a process of constructing an explanation needed to validate a mathematical claim. An example of a mathematical claim could be: *a subset of \mathbb{R} is closed iff it contains all its boundary points*. Proving this claim will involve constructing an explanation a justification that would lead to the conclusion that a subset of \mathbb{R} is closed if it contains all its boundary points. Constructing such an explanation can involve defining terms such as a boundary point and a closed set from Topology of the Real Line. Justifying mathematical assertions is central to the learning of mathematics and even making crucial decisions in everyday life.

Justifications have been classified as either pragmatic or conceptual justifications by Balacheff [19]. Balacheff [19] use the term pragmatic justifications to describe explanations based on use of particular instantiations in proving while conceptual justifications refer to abstract formulations of properties and relationships among pertinent mathematical ideas embedded in the conjecture whose truth-value a prover seeks to establish. A mathematical proof can thus be conceived as a product of the process of proving. So a mathematical proof is a product that results from mathematicians' attempts to establish the truth or falsity of a mathematical proposition as defined by Harel and Sowder [6]. From the discussion it can be observed that proving is a process of removing one's doubts about the accuracy (or lack thereof) of a mathematical claim.

When removing one's doubts about a conjecture a prover engages in activities that involve manipulating mathematical objects in some specific ways. Hence, proving can be viewed as a path followed in the process of converting a conjecture into mathematical fact [20]. A mathematical fact can be a theorem, a lemma, or some corollary to a theorem. A pertinent question one can ask in light of difficulties students face with proof and proving is: what sort of activities do students engage in during the creation of the path connecting the conjecture (point of departure) and the mathematical theorem (destination). It is important to observe that these activities are determined by the student's knowledge structures which are products of the student's thinking about mathematical proof.

Ersen [20] defines mathematical proving as a process of establishing the logical structure of pertinent mathematical ideas embedded in the mathematical proposition through deductive chains of reasoning in order to validate or refute a mathematical proposition. A limitation of Ersen [20] definition is that it is twisted in favour of deductive reasoning yet the process of proving can also call for counter-argumentation. So, one end of the deductive-inductive continuum of the proving process is heavily compromised. Hence, a prover who shares the same view of mathematical proof with Ersen [20] is more likely to use the axiomatic proof scheme even in proof situations that call for use of particular instantiations. Therefore, it can be noted that while a definition should be seen as a complete description of the structure of mathematical ideas pertinent to a concept being defined that capture all instances of that idea, we see that thinking of mathematical proof as chain of deductive reasoning is incomplete. Defining mathematical

proving as a sequence of deductive reasoning is incomplete because it does not encompass proof situations that call for proof method by, for example, refutation.

3. DISCUSSION

The foregoing review of literature justifies the importance of fostering a comprehensive conception of mathematical proof among students. Thus, mathematical proving should be viewed as a search for a deductive argument to validate a true mathematical proposition or the search for a counter-argumentation for the purpose of refuting a false conjecture. Stylianou *et. al.* [21] define counter-argumentation as the process of envisioning conditions, usually by picking instances that undermine the conjecture.

A mathematical proof should also be seen as a communicative act made within the mathematical community which ensures correctness of a given conjecture through the use of an analytic argument as proposed [22-23] posit that an analytic argument involves the application of both inductive and axiomatic reasoning aspects in the path of a proof. An axiomatic justification that supports a mathematical statement begins with some axioms, definitions, and previously proven theorems and then uses logically permissible rules of logic to draw a conclusion. On the other hand, an inductive argument is construed to involve referential mathematical objects. Referential objects include tables, graphs and other displays (usually in visual form) that are used to ensure correctness (or lack thereof) of proposition through the structural-intuitive mode of thought [23].

A structural-intuitive mode of thought involves examining a mathematical proposition to determine whether it is a consequence of mental models a prover associates with the mathematical ideas embedded in the mathematical conjecture being validated. In other words, a structural-intuitive mode of reasoning involves use of particular instantiations. For example, part of the process of proving the implication statement of the theorem: *A subset U of \mathbb{R} is open iff it can be expressed as a countable collection of disjoint open intervals in \mathbb{R}* , involves showing that two arbitrary intervals I_x and I_y selected from U are disjoint. The proving exercise can involve using a diagrammatic instantiation to demonstrate the fact that, $I_x \cap I_y = \emptyset$. Use of a diagrammatic instantiation together with axiomatic application during proving just described here illustrates the crucial interplay between axiomatic and inductive modes of reasoning involved in mathematical proof constructions.

To elucidate the point, we wish to make about the manner in which students should think of the notion of mathematical proof we focus our attention to the question: *what sort of activities should then characterise the path involved in mathematical proving?* Baki [24] describes three characteristics of the path followed when proving. First, there is the accuracy phase where the truth-value of an assertion is ascertained. Second, there is an illumination phase in which a prover explains why the proposition is accurate. Third, the proof once composed should then be abstracted by examining to see its application in other contexts. Next we address the question: *how should the concept of mathematical proof be understood by students in light of activities that constitute the path of a proof?* We draw on ideas Maya and Sumarmo [25] categorization of mathematical understanding.

Maya and Sumarmo [25] propose four levels of mathematical understanding ability: mechanical, inductive, rational, and intuitive understanding. Mechanical understanding occurs when a person memorizes rules and procedures that are then implemented correctly. However, no justification is provided by the individual as to why such rules and procedures lead to a conclusion. [16] write that inductive understanding of mathematical proof occurs when an individual verifies the accuracy of a statement by using mathematical objects (specific examples, diagrams) drawn from a proper subset of the set of objects to which the statement pertains.

Rational understanding of mathematical proof is defined as when an individual applies rules and procedures to establish the correctness of an assertion meaningfully, that is, application of such rules and procedures is accompanied by a justification. Finally, intuitive understanding is used to describe scenario in which a prover demonstrates an awareness of the truth of an assertion and has no doubts about its truth upon the production of the proof. In other words, an individual possessing intuitive understanding of a proof of mathematical proposition would have attained absolute conviction.

The connections between modes of thought by Weber and Mejia-Ramos [16], the activities of the proof path [24] and the categories of understanding of mathematical proof proposed by Baki [24] are now presented. Technical and symbolic manipulation of mathematical objects leads to mechanical understanding of mathematical claims. In this case the purpose of symbolic manipulations is to verify the accuracy of a mathematical assertion. With regard to the structural-intuitive mode of thought a prover verifies the accuracy of mathematical statement by using specific examples, and diagrams drawn from a proper subset of the set of objects to which the statement pertains.

It can be inferred that the structural-intuitive mode of thought will lead to inductive understanding. An example that can be used to illustrate the much desired interaction between axiomatic and structural-intuitive modes of reasoning about mathematical proof is the Archimedean principle in Real Analysis which states that *If $a, b \in \mathbb{R}$ with $a > 0$ then there exists a natural number n for which $na > b$* . The interpretation of the Archimedean principle is that the set of natural numbers \mathbb{N} is not bounded above.

To construct a proof of the Archimedean principle a prover can proceed in the following ways. First, a structural-intuitive verification can involve use of specific example $a = \frac{1}{2}, b = 3$ for which the inequality $\frac{1}{2}n > 3 \Rightarrow n > 6$. While the specific example verifies the Archimedean principle and helps to gain inductive understanding of the principle it does not count as proof. In other words, the single example employed cannot be elevated to the status of a mathematical proof because by virtue of being an empirical verification it does not provide conclusive evidence about the truth of the Archimedean principle.

Second, to compose proof of the Archimedean principle a prover can employ axiomatic reasoning or the structural thinking where a person draws on ideas such as the *Axiom of completeness* of \mathbb{R} as a real field. The proof method by contradiction could be employed to deduce that the set $S = \{ka : k \in \mathbb{N}\}$ has no least upper bound and hence S is not bounded above. Within the proof path a prover should provide justification of logical inferences made. In other words, a person engaging with the proof should justify why for instance a can be used in place of ε (epsilon radius). Further, if u is a least upper bound of S , a prover should explain why the relation $u - a < N$ holds and subsequently leads to a contradiction to the supposition that S is bounded above. Furnishing such finer details of the path will lead to rational understanding of the proof of the Archimedean principle. Constructing a mathematical proof in this manner corresponds to activities in the illumination phase of the path of a proof suggested by Baki [24]. It can be noted that a prover would have gone beyond the verification phase of proof construction.

Finally, the Archimedean theorem can be abstracted by examining it in order to follow the reasoning involved, and determine how the theorem can be coordinated with other theorems and lemmas to compose proofs of other mathematical theorems. For example, the Archimedean principle can be examined to see its application in proving theorems in Real Analysis. For instance, the Archimedean principle can be used in conjunction with the axiom of completeness to prove the rational density theorem in \mathbb{R} which is stated as: let $x, y \in \mathbb{R}$ with $x < y$ then there is a rational number, r , such that $x < r < y$.

Furthermore, in order to develop a comprehensive view of mathematical proof among students it is important that they attain intuitive understanding of mathematical proofs composed. As alluded to earlier, intuitive understanding is said to have been developed when an individual has absolute conviction about the truth of a mathematical proposition. Weber [23] have revealed that students have instead shown some relative conviction in proofs they would have generated. For example, a study by Harel [9] revealed that students exhibited relative conviction in proofs constructed in the following manner. After producing deductive arguments to validate a mathematical conjecture that requires proof by axiomatic reasoning, the students went on to verify the same conjecture using specific numeric examples. In other words, the students were not aware that valid deductive justifications provide complete and conclusive evidence about the truth of the conjecture and so such cases do not warrant further empirical verifications.

4. CONCLUSION

This piece has discussed literature on students' understanding schemata with regard to the notion of mathematical proof. The construct of a concept understanding schemata coined [25] refers to ways in which an individual uses a particular concept in sense making of ideas embedded in the concept as well as how that concept is applied in problem solving contexts. As noted earlier the concept of mathematical proof is central to mathematical learning because it provides a justification for the truth or falsity of a mathematical statement. Our interrogation of literature has identified an analytic conception of the idea of mathematical proof as more powerful. Studies examined have revealed that students have difficulty in acquiring rational and intuitive understanding of mathematical proof. Further, studies have revealed that students have displayed relative conviction in proofs of mathematical statements formulated via deductive means by engaging in extra empirical verifications of the statements. Overall, literature examined has shown lack of deep understanding of the notion of mathematical proof among students. Evidence of fragile grasp of the concept of mathematical proof includes rote memorization of uncoordinated fragments of proof facts to be regurgitated later. On the basis of these students' understandings of mathematical proof we recommend that:

- a. Preferably an analytic conception of mathematical proof should be acquired by students. An analytic understanding of mathematical allows interplay between the axiomatic and structural-intuitive modes of thought. So, one of the goals of current research efforts in mathematics education concerning the concept of mathematical proof should aim at promoting an analytic conception of

mathematical proof. This is so because challenges faced by students in composing mathematical proofs can only be overcome if a comprehensive view of mathematical proof is achieved among students.

- b. Efforts intended to understand critical elements in students' thinking processes should also involve evaluating whether a comprehensive conception of the definition of mathematical proof would have been developed.

The *voice* of the students should be the focal idea in current research endeavours to build a genuine and authentic analytic conception of mathematical proof among student teachers. Hence, efforts to promote a comprehensive conception of mathematical proof should be based on students' own proof construction productions as opposed to a focus on rote learning of instructors' notes.

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