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Development of primary school fourth-grade students' fraction calculation strategies through the argumentation method

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ABSTRACT

One of the most fundamental subjects of primary school mathematics, the concept of fraction plays a key role in students' educational background. Students experience difficulties and misconceptions regarding this concept. Therefore, it is of great importance to understand this subject well and to use different teaching methods and techniques. Various methods can be used in learning mathematics, such as invention method, computer-assisted teaching, problem solving, creative drama and discussion. Argumentation method is a scientific discussion of a claim on a subject by providing data, warrant and backing, and by revealing the rebuttal and the invalidity of the claim. Students gain more skills by interacting with teachers or collaborating with their peers, and that they possess skills that have not been obtained before or that are too difficult to achieve on their own. Therefore, the argumentation method was chosen for its effect on the development of primary school students' fraction calculation strategies. Having a case study design, this study was conducted with 28 primary school fourth-grade students and 10 open-ended questions prepared by the researchers were used during interviews. The results suggested that primary school students were able to develop fraction calculation strategies through the argumentation method. In general, inventing strategies, using representations, number sense, addition and subtraction, using unit fractions and proportional reasoning were used as fraction calculation strategies. It recommended to use the argumentation method in other critical subjects of mathematics as well.

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1. INTRODUCTION

Primary school mathematics is a course that prepares children for life and contributes greatly to their development; therefore, the objectives of this course are comprehensive. The aim of this course in the Primary School Mathematics Curriculum [1] is defined as "developing mathematical literacy skills and using these skills effectively, expressing their reasoning processes with their own thoughts, being aware of their friends' thinking gaps as well as realizing and complementing the mistakes in the reasoning processes". Various methods such as problem solving, creative drama and discussion appropriate for constructivist understanding can be used in learning mathematics to achieve these above-mentioned objectives. Argumentation is considered as one of these methods. Argumentation is a scientific discussion of a claim on a subject by providing data, warrant, and backing, and by revealing the rebuttal and the invalidity of the

claim. Toulmin's model is the most leading one even though many models can be used within the scope of the argumentation method. Toulmin's [2] argumentation model refers to the main components such as claim, data, backing and warrant, while rebuttal and qualifier are secondary components.

Figure 1 presents Toulmin [2] outlines. Data refer to the observations, proofs, event-facts and knowledge regarding the claim. These data include all the learning related to the situation that the individual has learned in advance or given in the content. Claims are thoughts and ideas put forward about a subject or event. Warrant provides a link between the claim and the data. It explains how the data support a given claim and include principles, rules and generalizations. Backing refers to strengthening the credibility of the warrants and claims through making the backings more acceptable. A qualifier is to limit the claim. It is also considered as phrases that draw the borders within a certain scope and distinguish them from different events and facts in terms of their qualities. Rebuttal specifies conditions when the claim is invalid [3].

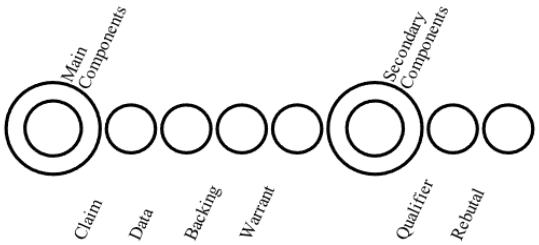

Figure 1.Toulmin argumentation model [2]

Pilar and Jimenez [4] defined argumentation method as using reasoning skills by choosing among different explanations as shown in Figure 2. Likewise, numerous researchers pointed out that the argumentation method contributes to reasoning skills [5]–[8]. The argumentation method ensures to understand a concept and pay attention while making sense of the concept and to build a bridge between prior experiences and new knowledge [9]. Similarly, mathematics learning involves consistency with already learned information, principles and newly learned ones [10]. Thus, it will be more effective and easier for students to remember their prior knowledge and bring them into the learning environment to create a schema for the new knowledge in their mind. Moreover, there will be interaction among students, and they will achieve better results since they work in larger groups [11]. Pilar and Jimenez [4] stated that students gain more skills by interacting with teachers or collaborating with their peers, and that they possess skills that have not been obtained before or that are too difficult to achieve on their own.

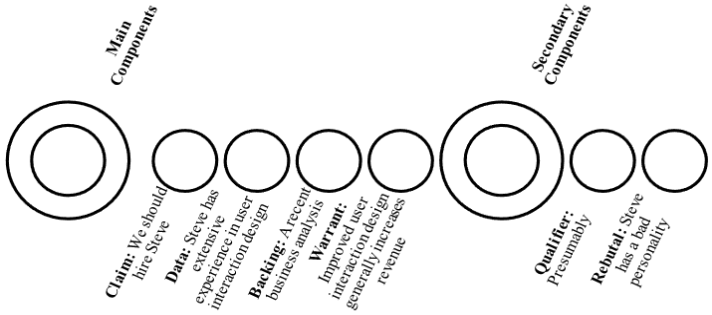

Figure 2. Example of discussion items in the model [2]

When students argue with their friends and are exposed to different ideas, they may reflect on their own ideas and those of others, helping them in eliminating their misconceptions and developing better learning [12]. Students learn to express their own ideas through working collaboratively [9], and to criticize and respect the ideas of their friends. Furthermore, their self-confidence develops and they become more socialized. The argumentation method aids in overcoming students' misconceptions about a subject and achieving satisfactory results in learning difficult subjects. Various studies revealed that learning fractions can be especially challenging for students [13]–[17].

In this regard, the use of the argumentation method in teaching fractions, which is amongst the challenging topics in mathematics, may help students and teachers. One of the most significant reasons why learning fractions is challenging for students is that they are different from whole numbers, and that fractions seem more abstract compared to other mathematics subjects. Considering that primary school students are in the concrete operational stage according to Ramos-Christian *et al*. [18], logical and concrete operations are required as much as possible while learning fractions. Visuals, objects and discussion on the topic can be used to make fractions concrete.

More aid can be obtained for discussing fractions and calculating strategies in fractions along with developing operational strategies through the argumentation method. It is likely that the argumentation method is troublesome for primary school students. However, given that one of the eight standards identified for mathematics in the Common Core State Standards for Mathematics [19] is the arguments and the others related to reasoning and criticism, it would be very appropriate for the students to gain these skills especially in primary school age. The description of this standard implicates that primary school students can construct arguments using concrete references such as drawings, objects and actions. These arguments can be logical and true even if they are not generalizations or certainties.

In the primary school mathematics curriculum, The Ministry of National Education emphasized that students' reasoning skills should be developed, they should realize the thinking gaps of their own and their friends, and that they should be able to fill these gaps [1]. Besides, it is of great importance for students to actively participate in the learning process and learn by doing and experiencing. The argumentation method covers all the components mentioned at this point. Therefore, it is remarkable to embody a subject that is difficult for students like fractions, and to actively participate in the learning process. This way of learning can also be probable with the argumentation method. This study is paramount in terms of revealing the implementation of the argumentation method at primary school level and the students' development of fraction calculation strategies.

Argumentation method is generally used in science studies around the world and in Turkey [20]–[35]. A limited number of studies were conducted on the use of argumentation method in the field of mathematics. Thus, this study is expected to not only fill the gap in the relevant literature but also show that the argumentation method can be used for both fractions and primary school students. In this context, the research question was related to how the argumentation method affected the primary school students' fraction calculation strategies. This study attempts to analyze the impact of the argumentation method on the development of primary school fourth-grade students' fraction calculation strategies.

2. RESEARCH METHOD

2.1. Research design

This study employed a case study design, one of the qualitative research methods, since the aim is to develop fraction calculation strategies through the argumentation method. The case study is an approach that allows an in-depth analysis to seek answers to scientific questions [36].

2.2. Working group

Case studies are generally conducted with a group of participants who engage with each other [37]. In-depth information can be gathered and a description can be made on the selected group. Therefore, this study was carried out with a class that interacted with each other. One of the random sampling methods, purposive sampling was used while determining the working group. Purposive sampling requires researchers to conduct an in-depth study depending on the goal of the study. The working group of the study consisted of 20 fourth-grade students learning at a primary school in the Eastern Mediterranean Region. The students were coded as S1, S2, S3… in order to comply with ethical rules.

2.3. Data collection tools and data collection process

One-on-one interviews were conducted with the students during data collection process. Interview is a data collection method that requires the interviewer to collect in-depth information through verbal communication or mass media that transmits audio and video instantly, like telephone [38]. This interview method provides the direct collection of data with the participants' responses and hence eliminating doubts. The researcher removes the question marks by asking the points that pop into his mind. In the present study, the researchers gathered direct and in-depth information through interviewing with a class group. The study deployed a worksheet developed by the researchers to develop primary school students' fractional strategies. The questions in the worksheet were submitted to two field experts for revision. Necessary corrections were made in line with the feedback. The worksheet includes 10 open-ended questions prepared in line with the primary school fourth-grade mathematics curriculum. Before proceeding with data collection, necessary

permissions were obtained from the university and the Ministry of National Education, further the data were collected by considering students' academic program. Students were initially informed about the argumentation method, and various examples were demonstrated in different fields. After observing that the students fully understood this subject, data were collected through the use of the worksheet on the subject of fractions.

2.4. Data analysis

In qualitative research, the data obtained from interviews, observations and documents are first coded and then synthesized by considering the coding, and the findings are presented [36]. Unlike quantitative research, not statistical analysis but description is mostly used in qualitative studies. Content analysis is one of the analysis methods used in qualitative research. The data are analyzed in depth and in detail, thus revealing new themes and codes. The data were gathered under the categories of "invented strategies, fraction representations, using number sense, addition and subtraction, proportional reasoning, using unit fractions". The data were stored directly and without any additions and transferred into the program to ensure the validity and reliability of the study. The consistency between intercoders was also checked for reliability. After the researchers completed the coding process, two more researchers who were experts in the field participated in the coding process and the results were compared by examining the consistency between intercoders. The consistency between the coders was calculated through using Miles and Huberman's formula [38] (reliability=agreement/agreement+disagreement X 100). The consistency was identified as 0.84, which is satisfactory since the result is close to or over 90% [36]. In addition, member checking was performed for exploring the credibility of the study. The papers were returned to participants to check for accuracy. Moreover, each phase of the study was explained to the students in detail for contributing to the credibility.

The researchers were unbiased and objective at all phases to ensure internal validity. They tried to increase the internal validity by taking notes and giving instructions to the students when necessary. Last but not least, experts' views were taken into account regarding the appropriateness of the questions. External validity is mostly related to the generalizability of the study results. It is inappropriate to make generalizations in qualitative studies, since qualitative research is conducted on a social event, and it is challenging to repeat the same social event for the same person, place and time [36]. All phases of the study were explained in detail to confirm external validity. Descriptions were made clear for having other researchers work with a group of participants with a similar subject and similar characteristics.

Toulmin's model focuses on documents with well-formed explicit arguments. To adapt to the informal arguments made in usability issue discussions, we started by using Toulmin's model directly to code the two problems. Based on our observations, we have made the following adjustments in order to reveal what kind of mistakes students make in fraction learning. First, in our initial analysis, we found it extremely difficult to demarcate Warrant and Support in issue discussions. We have made adjustments to avoid ambiguity. From these two components, we evaluated all statements of students that act as a bridge between concrete facts as explanations with warrant. Second, to accommodate the concise, informal style of argument in issue discussions, we have removed rejection as an argument component and instead set the point of view of any comment in the discussion as support or against with the original claim. Finally, we excluded the qualifier due to its low representation in our issue discussion data. These changes resulted in the components coding scheme used in content analysis.

It has been taken with the decision of Kahramanmaraş Sütçü İmam University Social and Human Sciences Ethics Committee dated 09.09.2021 and numbered 56553-44/8 that there is no obstacle in the implementation of the research. In addition, application permissions were obtained with the decision of Kahramanmaraş Provincial Directorate of National Education dated 15.09.2021 and numbered E-35776031-605.01-32015728.

3. RESULTS

This section presents findings on examining the development of primary school fourth-grade students' fraction calculation strategies through the argumentation method. Table 1 displays 28 students' scores related to the argumentation components. Students were required to write 5 components for each of the 10 questions on the worksheet. The maximum score that can be obtained from the worksheet is 50 with a total of 50 components, each of which is 1 point. S5 was found to have the highest score with 40, while S2 had the lowest score with 0. The most frequently repeated score is 7 with 7 repetitions. The reason why the students failed in writing the components may be because activities and lectures based on the argumentation method are not included in primary school instruction.

Table 1. The scores students got from the argumentation components

Student codes	Scores	Percentage (%)
S05	40	100
S17	24	60
S14	22	55
S08	22	55
S01	21	52,5
S12	19	47,5
S10	18	45
S07	17	42,5
S27	15	37,5
S26	14	35
S11	13	32,5
S09	13	32,5
S22	12	30
S19	11	27,5
S15	10	25
S16	9	22,5
S17	7	17,5
S18	7	17,5
S19	7	17,5
S20	7	17,5
S21	7	17,5
S22	7	17,5
S23	7	17,5
S24	6	15
S25	5	12,5
S26	2	5
S27	2	5
S28	0	0

Table 2 depicts the fraction calculation strategies and their use frequency by the students. The students were identified to mostly use the fraction representation strategy. All students used this strategy at least 2 times except for S18 and S16. This may be due to students' use of area model in fractions. The least used strategy was determined as using unit fractions. As is seen in Table 2, S4, S23 and S24 preferred this strategy. The reason why students used this strategy less often may be because they have a passing acquaintance with fractions. Upon examining Table 2, the majority of the students (except for S2, S8, S9, S11, S12, S15, S18, S22, S23 and S28) invented a strategy in calculating and used the proportional reasoning strategy (except for S2, S8, S18, S22, S23 and S28). Inventing strategies and using proportional reasoning strategy indicate the effect of the argumentation method since reasoning, creativity and higher-level thinking skills are used and developed effectively in the argumentation method.

Table 3 suggests that all of the students, except for S16 and S28, used the area model as a fraction representation. Students were noted to mostly use area model for fraction representation. This may arise from the fact that the fractions become more concrete with the use of area model. Besides, students frequently used the set model (except for S8, S9, S14, S18, S20, S25 and S28), yet it is not a model that students can use a lot. This may be because of the effectiveness of the argumentation method on the development of fraction calculation strategies. Number line was identified as the least used model in representing fractions. As in Table 3, almost half of the class (13 students) used this model as well. It may be wise to mention that the argumentation method has an impact on the use of the number line model.

Table 2. Strategies used by the students in calculating fractions

Strategies	Invented strategy	Fraction representation	Number sense	Addition and subtraction	Using unit fractions	Proportional reasoning	Total
S1	1	6	-	1	-	2	10
S2	-	2	-	-	-	-	2
S3	2	7	-	1	-	2	12
S4	2	6	-	1	1	2	12
S5	2	9	-	1	-	2	14
S6	1	4	-	1	-	1	7
S7	3	5	-	-	-	2	10
S8	-	2	1	1	-	-	4
S9	-	6	-	1	-	1	8
S10	2	6	-	2	-	2	12
S11	-	5	1	1	-	2	9
S12	-	4	1	3	-	2	10
S13	1	3	-	-	-	2	6
S14	3	8	-	2	-	2	15
S15	-	5	-	-	-	1	6
S16	1	1	-	-	-	2	2
S17	1	5	-	2	-	1	9
S18	-	1	1	2	-	-	4
S19	1	5	-	2	-	2	10
S20	2	2	-	-	-	2	6
S21	1	2	-	-	-	1	4
S22	-	2	1	-	-	-	3
S23	-	2	-	-	1	-	3
S24	1	2	-	1	1	1	6
S25	1	2	-	-	-	2	5
S26	1	3	-	2	-	1	7
S27	1	2	-	1	-	1	5
S28	-	-	-	1	-	-	1
Total	27	107	5	26	3	36	194

Table 3. Number of models used in fraction representation

Students	Area model	Set model	Number line model	Total
S5	2	7	1	10
S3	2	4	1	7
S9	6	-	1	7
S10	5	1	1	7
S2	1	1	-	2
S6	1	2	-	3
S7	3	1	1	5
S8	1	-	1	2
S4	4	1	1	6
S1	4	1	1	6
S11	3	1	1	5
S12	2	1	1	4
S13	2	1	-	3
S14	7	-	1	8
S15	4	1	-	5
S16	-	1	-	1
S17	4	1	-	5
S18	1	-	-	1
S19	3	1	1	5
S20	2	-	-	2
S21	1	1	-	2
S22	1	1	1	3
S23	1	1	-	2
S24	1	1	-	2
S25	2	-	-	2
S26	2	1	-	3
S27	1	1	-	2
S28	-	-	-	0
Total	66	31	13	110

Figure 3 displays that the student coded S7 solved the question by drawing the remaining three cakes and the strawberry slices on them, since it was said that there were four cakes in the question. In this regard, as the student can see the strawberry slices as a whole, it is easier to understand and solve the question. As can be seen in Figure 4, S11 wrote the claim and data components while solving the question. Drawing a picture while writing the data made it easier for the student to understand. The student drew 12

stars for the rest of the set. The student's use of claim and data components may have helped him/her understand and solve the question.

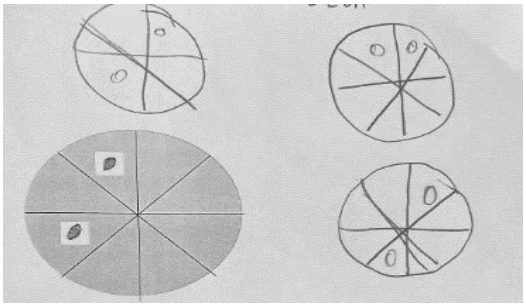

Figure 3. An example of area model

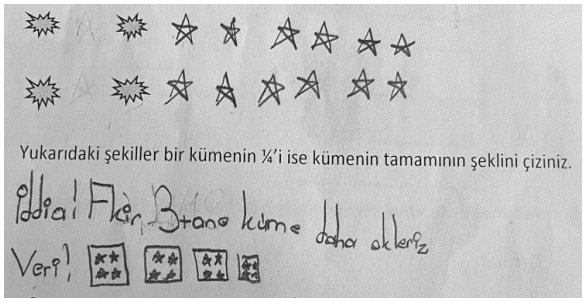

Figure 4. An example of set model

Figure 5 demonstrates that the student coded S22 noted the claim and data components while solving the question. Afterwards, the student drew the number line and assigned each unit as two meters. Upon analyzing Figure 6, the student coded S10 invented the strategy of smashing the cookies instead of dividing. First, the student divided three cookies from two pieces into a total of six pieces, then solved the question by dividing the last cookie into six equal pieces.

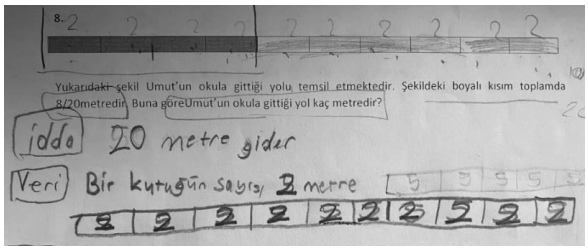

Figure 5. An example of number line model

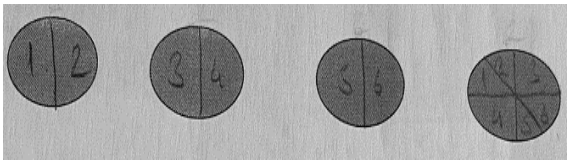

Figure 6. An example of invented strategy

Figure 7 illustrates that the student coded S11 started to solve the question by writing down the data and claim. Making operations by drawing a figure in the data section contributed to solving the question. As is seen in Figure 8, S1 first wrote down the fractions that represent the portion of objects in the figures and then solved the question by creating the problem statement.

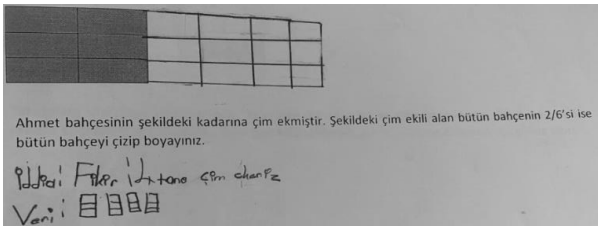

Figure 7. An example of number sense strategy

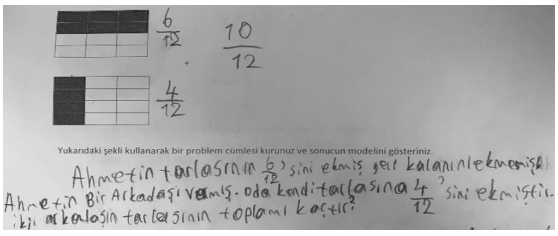

Figure 8. An example of addition and subtraction strategy

4. DISCUSSION

This study attempts to develop the primary school students' fraction calculation strategies through the argumentation method. In this vein, the results revealed that the students' fraction calculation strategies

developed through the argumentation method. In line with the findings, the students were determined to mostly use the fraction representation strategy. Among the fraction representation strategies, all but two students were found to use the area model. Yet, the students did not prefer the number line-length model as much as the other models. A similar result emerged in the study conducted by Duran [39] with pre-service secondary mathematics teachers. The reason why students mostly prefer the area model may be because this model is more concrete and visual compared to the others. Duran [39] concluded that the area model was mostly used as it requires equal separation. At that point, we do not agree with Duran [39] since the set model and the number line model also necessitate equal separation. Similarly, Doğan and Tertemiz [40] also noted that the number line model was used less frequently than the area and set model. The results implicated that the set model was frequently preferred by the students, except for few. In Doğan and Paydar's study [41], it was found that the classroom teachers predominantly showed the area model while showing fractions. Another study carried out by Cumhur and Korkmaz [42] suggested that pre-service primary school teachers mostly used the number line model, while the set model less. This may result from the fact that their abstract thinking and high-level thinking skills are developed by their age.

Duatepe *et al*. [43] outlined that the questions should be prepared by using different strategies regardless of the traditional understanding in order to improve students' proportional reasoning skills. This result is in line with those of the present study on the grounds that all but six of the students solved the questions through using the proportional reasoning strategy.

The results also revealed that most of the students used the addition and subtraction strategy. Contrary to popular belief, addition and subtraction in fractions are challenging for students. Because students were determined to perceive fractions as natural numbers while using adding and subtracting operations in fractions, and by considering the numerator and the denominator separately [44]. The present study emphasized that the use of the argumentation method might be the reason why students did not make similar mistakes while adding and subtracting in fractions.

Çekirdekçi *et al*. [44] defined number sense as the ability to solve problems encountered in daily life, to name numbers and to include strategies such as estimation and mental calculation. Several studies affirmed that number sense is quite low among primary school fourth-grade students [45], [46]. In this regard, the finding of the study related to number sense is congruent with those of other studies. Çekirdekçi *et al*. [45] interpreted the reason for the lower level of number sense as such: "The lower success of students' number sense and the students' tendency towards using rule-operation based methods may be the result of the activities in the Mathematics Curriculum and textbooks used in our country."

Besides, fewer students were identified to use the unit fraction strategy. Divrik and Pilten [47] concluded that the students could not easily determine the unit fraction, inasmuch as they did not have any conceptual knowledge about the unit fraction. The findings of the study are similar to these findings.

Inventing a strategy requires creative thinking. The results demonstrated that most of the students were able to invent a strategy. It is referring to the students' creative thinking and the positive effect of the argumentation method.

A scientific discussion, the argumentation method is generally used in science studies in the world and Turkey [48]–[52]. A limited number of studies were conducted on the argumentation method in other fields. Çelik *et al*. [53] examined the development of pre-service mathematics teachers' critical reading levels through on-line argumentation method, and concluded that their critical reading levels increased significantly. This paves the way for the fact that the argumentation method is effective when used in other fields apart from natural sciences.

Uc and Benzer [54] investigated the effect of argumentation practices performed with writing activities on the secondary school students' creative writing and concept learning. The results revealed a significant positive difference across students' creative writing in the experimental group. Kara *et al*. [24], Ecevit *et al*. [55], Karakaş and Sarıkaya [56] pinpointed the positive effect of the argumentation method at primary school level. Further, they recommended disseminating the use of this method in primary schools.

5. CONCLUSION

The study involved such fraction calculation strategies as inventing strategies, using fraction representation, number sense, addition and subtraction, using unit fraction and proportional reasoning. The results of this study also highlighted that the students developed fraction calculation strategies through the argumentation method, still they had difficulty in writing the argumentation components. This may arise from the lack of using argumentation method at primary school level. Because the argumentation method at primary school level can be high in terms of mental performance. It will require metacognitive performance to think of many components together and to build bases for each of them.

Another result of the study suggested that participants rarely used the number sense. Number sense is more strategically important at the primary school level. Many different notations and manipulatives must

be used to develop number sense. One of them can be the argumentation method. Among the results of this study, it is important that the argumentation method has a positive effect on the sense of number. My argumentation method is widely used at secondary school level, as it enables children to develop different perspectives, use different representations, and enable group work. It is seen that lowering this grade level to primary school is also effective in students' mental abstract-concrete concept relations.

In consequence, this study maintained that the fourth-grade students' fraction calculation strategies improved through the argumentation method. As the lessons, activities and plans based on the argumentation method become widespread. The components of this method will be better understood by the students and thus, it will become much more efficient. The argumentation method should be more preferred by teachers in primary school lessons. Studies based on argumentation method should be increased in the field of mathematics education as in science education. Argumentation method can also be included in order to develop students' sense of number. More studies should be carried out through using argumentation method at primary school level. Similar studies may examine the effect of argumentation method on a different subject of fractions.

REFERENCES

[1] MEB, "Matematik Dersi Öğretim Programi," 2018. http://mufredat.meb.gov.tr/ProgramDetay.aspx?PID=329.

[2] S. Toulmin, *The uses of argument*. New York: Cambridge Unıversıty Press, 2003.

[3] S. Erduran, S. Simon, and J. Osborne, "Tapping into argumentation: Developments in the application of Toulmin's Argument Pattern for studying science discourse," *Science Education*, vol. 88, no. 6, pp. 915–933, Nov. 2004, doi: 10.1002/sce.20012.

[4] M.-P. Jime´nez-Aleixandre, "Knowledge producers or knowledge consumers? Argumentation and decision making about environmental management," *International Journal of Science Education*, vol. 24, no. 11, pp. 1171–1190, Nov. 2002, doi: 10.1080/09500690210134857.

[5] Ö. G. D. H. Aldağ, "Toulmın tartışma modeli," *Çukurova Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, vol. 15, no. 2, pp. 13–33, 2006.

[6] Z. Aydoğdu, "Investigation of effects the argumentation based science teaching on the academic success, motivation, interest and attitudes towards science," Sakarya University Institute of Educational Sciences, 2017.

[7] R. Driver, P. Newton, and J. Osborne, "Establishing the norms of scientific argumentation in classrooms," *Science Education*, vol. 84, no. 3, pp. 287–312, May 2000, doi: 10.1002/(SICI)1098-237X(200005)84:3<287::AID-SCE1>3.0.CO;2-A.

[8] A. Walker, and T. Kettler, "Developing critical thinking skills in high ability adolescents: Effects of a debate and argument analysis curriculum," *Talent*, vol. 10, no. 1, pp. 21–39, Oct. 2020, doi:10.46893/talent.758473.

[9] L. K. Berland and B. J. Reiser, "Classroom communities' adaptations of the practice of scientific argumentation," *Science Education*, vol. 95, no. 2, pp. 191–216, Mar. 2011, doi: 10.1002/sce.20420.

[10] K. Inagaki, G. Hatano, and E. Morita, "Construction of mathematical knowledge through whole-class discussion," *Learning and Instruction*, vol. 8, no. 6, pp. 503–526, Dec. 1998, doi: 10.1016/S0959-4752(98)00032-2.

[11] N. M. Webb, "Task-Related Verbal Interaction and Mathematics Learning in Small Groups," *Journal for Research in Mathematics Education*, vol. 22, no. 5, pp. 366–389, Nov. 1991, doi: 10.5951/jresematheduc.22.5.0366.

[12] D. Cross, G. Taasoobshirazi, S. Hendricks, and D. T. Hickey, "Argumentation: A strategy for improving achievement and revealing scientific identities," *International Journal of Science Education*, vol. 30, no. 6, pp. 837–861, May 2008, doi: 10.1080/09500690701411567.

[13] M. Aksu, "Student Performance in Dealing With Fractions," *The Journal of Educational Research*, vol. 90, no. 6, pp. 375–380, Jul. 1997, doi: 10.1080/00220671.1997.10544595.

[14] Ç. Kılıç and A. Özdaş, "Ilköğretim 5. Sin Öğrencilerinin Kesirlerde Karşilaştirma Ve Siralama Yapmayi Gerektiren Problemlerin Çözümlerinde Kullandiklari Temsiller," *Kastamonu Education Journal İlköğretim 5. Sinif Öğrencilerinin Kesirlerde*, vol. 18, no. 2, pp. 513–530, 2010, [Online]. Available: https://dergipark.org.tr/en/pub/kefdergi/626009.

[15] J. Namkung and L. Fuchs, "Remediating Difficulty with Fractions for Students with Mathematics Learning Difficulties," *Learning Disabilities: A Multidisciplinary Journal*, vol. 24, no. 2, pp. 36–48, 2019, doi: 10.18666/LDMJ-2019-V24-I2-9902.

[16] Z. Çakmak gürel and M. Okur, "Ortaokul 6. ve 7. Sınıf Öğrencilerinin Kesirler Konusundaki Kavram Yanılgıları," *Erzincan Üniversitesi Eğitim Fakültesi Dergisi*, vol. 18, no. 2, Dec. 2016, doi: 10.17556/jef.30116.

[17] D. Zhang and F. D. Rivera, "Predetermined accommodations with a standardized testing protocol: Examining two accommodation supports for developing fraction thinking in students with mathematical difficulties," *The Journal of Mathematical Behavior*, vol. 62, p. 100861, Jun. 2021, doi: 10.1016/j.jmathb.2021.100861.

[18] V. Ramos-Christian, R. Schleser, and M. E. Varn, "Math fluency: Accuracy versus speed in preoperational and concrete operational first and second grade children," *Early Childhood Education Journal*, vol. 35, no. 6, pp. 543-549. Feb. 2008. doi:10.1007/s10643-008-0234-7.

[19] Common Core State Standards Initiative, "Common Core State Standards for Mathematics," 2010. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf.

[20] Z. Apaydin and M. A. Kandemir, "Opinions of Classroom Teachers about the Use of Argumentation Method in Science Classroom in Primary School," *Journal of Computer and Education Research*, vol. 6, no. 11, pp. 106–122, Apr. 2018, doi: 10.18009/jcer.387033.

[21] H. Aktamış and E. Higde, "Fen eğitiminde kullanılan argümantasyon modellerinin değerlendirilmesi," *Mehmet Akif Ersoy Üniversitesi Eğitim Fakültesi Dergisi*, vol. 35, pp. 136–172, 2015.

[22] E. Aymen Peker, Z. Apaydın, and E. Taş, "Isı yalıtımını argümantasyonla anlama: İlköğretim 6. sınıf öğrencileri ile durum çalışması," *Dicle Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, vol. 4, no. 8, pp. 79–100, 2012.

[23] Ş. Uluçinar sağir, Ü. İ. Soylu, and A. Bolat, "Determination of Argumentation Levels of 7th Grade Students In Power and Energy Unit," *Anadolu Journal Of Educational Sciences International*, pp. 184–203, Jan. 2021, doi: 10.18039/ajesi.726305.

[24] S. Kara, S. Yilmaz, and S. Kingir, "Argümantasyon Tabanlı Bilim Öğrenme Yaklaşımının İlkokul Öğrencilerinin Akademik Başarılarına ve Argümantasyon Kalite Düzeylerine Etkisi," *Kastamonu Eğitim Dergisi*, May 2020, doi: 10.24106/kefdergi.3785.

[25] T. Ecevit and F. Kaptan, "Describing the argument based inquiry teaching model designed for gaining the 21st century skills," *Hacettepe Egitim Dergisi*, vol. 36, no. 2, pp. 470–488, 2021, doi: 10.16986/HUJE.2019056328.
[26] S. Okumuş, "Argümantasyon Destekli İşbirlikli Öğrenme Modelinin Akademik Başarıya, Eleştirel Düşünme Eğilimine ve Sosyobilimsel Konulara Yönelik Tutuma Etkisi," *Ondokuz Mayıs Üniversitesi Eğitim Fakültesi Dergisi*, vol. 39, no. 2, pp. 269–293, 2020, doi: https://doi.org/10.7822/omuefd.570419.
[27] A. Cirit Gül, Z. Apaydın, E. O. Çobanoğlu, and P. Tağrikulu, "Fen öğretiminde toulmin argümantasyon modelinin sınıf dışı (outdoor) eğitim süreci ile bütünleştirilmesi: örnek etkinlikler," *Türkiye Bilimsel Araştımalar Dergisi*, vol. 3, no. 2, 2018.
[28] C. Ulu, "Fen Bilimlerinde Öğrencilerin Oluşturdukları Argümanların Kalitesi ile Kavram Öğrenme Seviyeleri Arasındaki İlişki," *International Journal of Science and Education*, vol. 1, no. 1, pp. 11–24, [Online]. Available: http://dergipark.gov.tr/ubed.
[29] T. Long, G. Su, Z. Wang, and Q. Zeng, "The Effect of Thinking Tools on the Argumentation Skills of Pre-Service Science Teachers in Flipped Learning," in *2020 International Symposium on Educational Technology (ISET)*, Aug. 2020, pp. 48–52, doi: 10.1109/ISET49818.2020.00020.
[30] H. Kartika, M. T. Budiarto, and Y. Fuad, "Argumentation in K-12 Mathematics and Science Education: A Content Analysis of Articles," *International Journal of Research in Education and Science*, vol. 7, no. 1, p. 51, Dec. 2020, doi: 10.46328/ijres.1389.
[31] H. Ha and H.-B. Kim, "Framing Oneself and One Another as Collaborative Contributors in Small Group Argumentation in a Science Classroom," *International Journal of Science and Mathematics Education*, vol. 19, no. 3, pp. 517–537, Mar. 2021, doi: 10.1007/s10763-020-10071-z.
[32] P.-S. Hsu, S. Mukhopadhyay, and R. Al-Ararah, "Exploring current practice of using technology to support collaborative argumentation in science classrooms.," *Middle Grades Review*, vol. 6, no. 1, p. Article 6, 2020.
[33] I. L. L. Ping, L. Halim, and K. Osman, "Explicit Teaching of Scientific Argumentation as An Approach in Developing Argumentation Skills, Science Process Skills and Biology Understanding," *Journal of Baltic Science Education*, vol. 19, no. 2, pp. 276–288, Apr. 2020, doi: 10.33225/jbse/20.19.276.
[34] S. Erduran, L. Guilfoyle, and W. Park, "Science and Religious Education Teachers' Views of Argumentation and Its Teaching," *Research in Science Education*, vol. 52, no. 2, pp. 655–673, Apr. 2022, doi: 10.1007/s11165-020-09966-2.
[35] H. Karakaş and R. Sarikaya, "Çevre-Enerji Konularına Yönelik Gerçekleştirilen Argümantasyon Temelli Öğretimin Sınıf Öğretmeni Adaylarının Argüman Oluşturabilmelerine Etkisi," *Pamukkale University Journal of Education*, vol. 48, pp. 346–373, Jan. 2020, doi: 10.9779/pauefd.524850.
[36] F. Büyüköztürk, Ş., Çakmak-Kılıç, E., Akgün, Ö.E., Karadeniz, Ş., & Demirel, *Bilimsel Araştırma Yöntemleri*. Ankara: Pegem Akademi, 2012.
[37] N. Karasar, *Bilimsel araştırma yöntemi: Kavramlar ilkeler teknikler*. Ankara: Nobel, 2019.
[38] Miles, M., B. and A. M. Huberman, *Qualitative data analysis: An expanded sourcebook*, 2nd ed., vol. 19, no. 1. Calif: SAGE Publications, 1994.
[39] N. B. Duran, "Ortaokul matematik öğretmen adaylarının alan ve pedagojik alan bilgileri çerçevesinde kesirlerle çarpma ve bölme işlemlerinin öğretimine ilişkin kullandıkları modeller," Pamukkale University Institute of Educational Sciences, 2017.
[40] A. Doğan and N. Işik Tertemiz, "Fraction models used by primary school teachers," *İlköğretim Online*, pp. 1888–1901, Sep. 2020, doi: 10.17051/ilkonline.2020.762538.
[41] A. Doğan and S. Paydar, "Üstün Yetenekli Öğrenciler ile Akranlarının Sayı Hissi Alt Bileşenlerinin Karşılaştırılması," *Kahramanmaraş Sütçü İmam Üniversitesi Sosyal Bilimler Dergisi*, vol. 17, no. 1, pp. 21–44, Apr. 2020, doi: 10.33437/ksusbd.689146.
[42] F. Cumhur and E. Korkmaz, "Sınıf Öğretmeni Adaylarının Kesirlerde Toplama İşlemi İle İlgili Öğretim Stratejilerinin İncelenmesi," *Anemon Muş Alparslan Üniversitesi Sosyal Bilimler Dergisi*, vol. 8, no. 6, pp. 1841–1854, Dec. 2020, doi: 10.18506/anemon.765378.
[43] A. Duatepe, O. Akkuş-Çıkla, and M. Kayhan, "Orantısal Akıl Yürütme Gerektiren Sorularda Öğrencilerin Kullandıkları Çözüm Stratejilerinin Soru Türlerine Göre Değişiminin İncelenmesi," *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, vol. 28, pp. 73–81, 2005.
[44] H. Önal and A. Yorulmaz, "İlkokul Dördüncü Sınıf Öğrencilerinin Kesirler Konusunda Yaptıkları Hatalar," *Eğitim ve Toplum Araştırmaları Dergisi*, vol. 4, no. 1, pp. 98–113, 2017.
[45] S. Çekirdekci, "4. Sınıf Öğrencilerinin Kullandıkları Sayı Hissi Stratejilerinin Belirlenmesi," *Social Mentality and Researcher Thinkers Journal*, vol. 6, no. 31, pp. 680–695, Jan. 2020, doi: 10.31576/smryj.512.
[46] D. Can, "İlkokul Dördüncü Sınıf Öğrencilerinin Sayı Duyusu Performansının Bazı Değişkenler Açısından İncelenmesi," *İlköğretim Online*, vol. 18, no. 4, pp. 1751–1765, Dec. 2019, doi: 10.17051/ilkonline.2019.639317.
[47] R. Divrik and P. Pilten, "Analysis of the primary school 3rd graders' errors on fractional numbers in terms of the unit fraction, symbol, and model," *Eğitim Kuram ve Uygulama Araştırmaları Dergisi*, vol. 7, no. 1, pp. 62–73, 2021.
[48] F. Böttcher and A. Meisert, "Argumentation in Science Education: A Model-based Framework," *Science & Education*, vol. 20, no. 2, pp. 103–140, Feb. 2011, doi: 10.1007/s11191-010-9304-5.
[49] L. A Bricker and P Bell, "Conceptualizations of argumentation from science studies and the learning sciences and their implications for the practices of science education," *Science Education*, vol. 92, no. 3, pp. 473–498, 2008.
[50] C. Tippett, "Argumentation: The language of science," *Journal of Elementary Science Education*, vol. 21, no. 1, pp. 17–25, 2009, doi: 10.1007/bf03174713.
[51] E. H. Rıdvan özcan, Hilal Aktamiş, "Fen Bilimleri Derslerinde Kullanılan Argümantasyon Düzeyinin Belirlenmesi," *Pamukkale Universitesi Egitim Fakultesi Dergisi-Pamukkale University Journal of Education*, vol. 43, pp. 93–106, 2018, [Online]. Available: https://dergipark.org.tr/en/download/article-file/401273.
[52] R. Demirel and H. Özcan, "Argümantasyon Destekli Fen Ve Mühendislik Uygulamalarinin 7. Sinif Öğrencilerinin Işik Konusuna Yönelik Başarilarina Etkisi," *Aksaray Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, vol. 5, no. 1, pp. 100–111, Jun. 2021, doi: 10.38122/ased.912508.
[53] T. Çelik, S. Gökçe, A. A. Yenmez, and İ. Özpınar, "Online argumentation: Perception of self-efficacy at critical reading," *Journal of Language …*, vol. 3, no. 2, pp. 117–134, 2017, [Online]. Available: https://www.ceeol.com/search/article-detail?id=577201.
[54] F. B. Uc and E. Benzer, "The effects of argumentation applications conducted with writing activities on the creative writing and concept learning of second school students," *Akdeniz Üniversitesi Eğitim Fakültesi Dergisi*, vol. 4, no. 1, pp. 79–104, 2021.
[55] T. Ecevit, N. Balci, M. Yildiz, and B. S. Sayan, "İlkokul Düzeyindeki Araştırma-Sorgulama, Argümantasyon ve STEM Temelli Uygulamalarının Tematik İçerik Analizi," *Yuzunci Yil Universitesi Egitim Fakultesi Dergisi*, vol. 18, no. 1, pp. 1100–1129, Jul. 2021, doi: 10.33711/yyuefd.957395.
[56] H. Karakaş, and R. Sarıkaya, "The effect of argumentation-based teaching performed for environment-energy issues on critical thinking of prospective classroom teachers," *İlköğretim Online*, vol. 19, no. 1, pp. 366–383, Jan. 2020, doi: 10.17051/ilkonline.2020.661855.

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